DIRECT STRENGTH METHOD DESIGN GUIDE CF06-1

design guide

Direct Strength Method (DSM) Design Guide

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JUNE 2006

Committee on Specifications
for the Design of Cold-Formed
Steel Structural Members

American Iron and Steel Institute

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The following Design Guide has been developed under the direction of the American Iron and Steel Institute Committee on Specifications for the Design of Cold-Formed Steel Structural Members. The development of the Guide was sponsored by the American Iron and Steel Institute. The AISI Committee on Specifications wishes to acknowledge and expresses gratitude to Dr. Benjamin Schafer of Johns Hopkins University who was the author of this Guide.

With anticipated improvements in understanding of the behavior of cold-formed steel and the continuing development of new technology, this material might become dated. It is possible that AISI will attempt to produce updates of this Guide, but it is not guaranteed.

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Preface

The Direct Strength Method is an entirely new design method for cold-formed steel. Adopted in 2004 as Appendix 1 to the *North American Specification for the Design of Cold-Formed Steel Structural Members,* this Guide provides practical and detailed advice on the use of this new and powerful design method. Features of the Guide include:

Design examples: Extensive design examples, with thorough commentary, covering 14 different cold-formed steel cross-sections under a variety of different loading and boundary conditions are provided (Chapter 8). The bulk of the design examples are based on the AISI (2002) *Cold-Formed Steel Design Manual* and allow engineers to make direct comparison between the Direct Strength Method and conventional design.

Tutorial: Introductory material to help engineers interpret elastic buckling analysis results, the heart of the Direct Strength Method, is provided (e.g., see Figure 2).

Charts: Prescriptive guidelines (Chapter 4) and an example (Section 8.13) for developing beam charts using the Direct Strength Method are provided. Similar examples are given for column charts – together they can be used to create span and load tables based on the Direct Strength Method.

The finer points: Details are not skipped over, for example, extensive discussion on how to handle unique situations in the elastic buckling analysis of members is provided (Section 3.3).

Acknowledgments

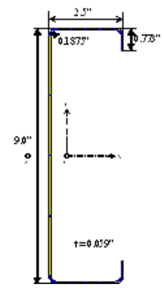
Funding for the development of this Guide was provided to Dr. Benjamin Schafer at Johns Hopkins University by the American Iron and Steel Institute. Members and friends of the American Iron and Steel Institute Committee on Specifications for the Design of Cold-Formed Steel Structural Members provided useful feedback in the development of this Guide. In particular, Helen Chen, Bob Glauz, Perry Green, Dick Kaehler, and Tom Miller provided comments that greatly improved the final version.

Design Examples *Quick Start*

This lists the six design examples provided in Section 8.1. Each design example uses the same cross-section, in this case, C-section 9CS2.5x059. Reference to AISI (2002) *Design Manual* examples is also provided.

8.1 C-section with lips

Given:
a. Steel: F_y = 55 ksi
b. Section 9CS2.5x059 as shown to the right
c. Finite strip analysis results (Section 3.2.1)
Required:
1. Flexural strength for a fully braced member
2. Flexural strength for L=56.2 in. (AISI 2002 Example II-1)
3. Effective moment of inertia
4. Compressive strength for a fully braced member
5. Compressive strength at F_n=37.25 ksi (AISI 2002 Example III-1)
6. Beam-column design strength (AISI 2002 Example III-1)



":=" vs. "="

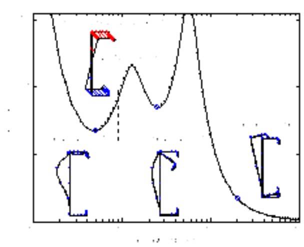
":=" in Mathcad, this ":=" symbol is how equations are defined. The right hand side is evaluated and the answer assigned to the left hand side. In this example $M_{cr\ell}$ is defined as $0.67M_y$ and then evaluated.

"=" in Mathcad, the "=" symbol is simply a print statement. In this example $M_{cr\ell}$ is defined as $0.67M_y$ with the ":=" symbol and its value, 85 kip-in., is printed to the screen with the "=" symbol.

8.1-1 Flexural strength for a fully braced member (AISI 2002 Example I-8)

Determination of the nominal flexural strength for a fully braced member is equivalent to determining the effective section modulus at yield in the main *Specification*. see AISI (2002) example I-8.

Finite strip analysis of 9CS2.5x059 in pure bending as summarized in Example 3.2.1
Inputs from the finite strip analysis include:



$M_y := 126.55\,kip \cdot in$

$M_{crl} := 0.67 \cdot M_y$ $\quad M_{crl} = 85\,kip \cdot in$

$M_{crd} := 0.85 \cdot M_y$ $\quad M_{crd} = 108\,kip \cdot in$

per DSM 1.2.2, M_n is the minimum of M_{ne}, $M_{n\ell}$, M_{nd}. For a fully braced member lateral-torsional buckling will not occur and thus $M_{ne} = M_y$, $M_{n\ell}$ and M_{nd} must still be checked.

$M_{ne} := M_y$ $\quad M_{ne} = 127\,kip \cdot in$ (fully braced)

Equation numbers refer to the relevant parts of DSM (Appendix 1 AISI 2004)

Local buckling check per DSM 1.2.2.2

if-then

"|" in Mathcad, the "|" symbol is for if-then statements. In this example if $\lambda_\ell \le 0.776$ $M_{n\ell}$ is M_{ne}, otherwise if $\lambda_\ell > 0.776$ then the second expression applies. The vertical bar shows the potential choices for $M_{n\ell}$.

$\lambda_l := \sqrt{\frac{M_{ne}}{M_{crl}}}$ $\quad \lambda_l = 1.22$ (subscript "l" = "ℓ") (Eq. 1.2.2-7)

$$M_{nl} := \begin{cases} M_{ne} & \text{if } \lambda_l \le 0.776 \\ \left[1 - 0.15 \cdot \left(\frac{M_{crl}}{M_{ne}}\right)^{0.4}\right] \left(\frac{M_{crl}}{M_{ne}}\right)^{0.4} \cdot M_{ne} & \text{if } \lambda_l > 0.776 \end{cases}$$ (Eq. 1.2.2-5) (Eq. 1.2.2-6)

$M_{nl} = 94\,kip \cdot in$

Distortional buckling check per DSM 1.2.2.3

units

In Mathcad, the solution includes units. Since M_y is given units of kip-in. and M_{crd} is defined in terms of M_y, M_{nd} is also in kip-in. In the program units can be changed and the results will modify accordingly.

$\lambda_d := \sqrt{\frac{M_y}{M_{crd}}}$ $\quad \lambda_d = 1.08$ (Eq. 1.2.2-10)

$$M_{nd} := \begin{cases} M_y & \text{if } \lambda_d \le 0.673 \\ \left[1 - 0.22 \cdot \left(\frac{M_{crd}}{M_y}\right)^{0.5}\right] \left(\frac{M_{crd}}{M_y}\right)^{0.5} \cdot M_y & \text{if } \lambda_d > 0.673 \end{cases}$$ (Eq. 1.2.2-8) (Eq. 1.2.2-9)

$M_{nd} = 93\,kip \cdot in$

Predicted flexural strength per DSM 1.3

min

"min" in Mathcad, the "min" function operates on variables in a row vector, and in this case provides the member strength M_n.

$M_n := \min\left(\left(M_{ne}\ \ M_{nl}\ \ M_{nd}\right)\right)$ $\quad M_n = 93\,kip \cdot in$

The geometry of this section falls within the "pre-qualified" beams of DSM 1.1.1.2 and the higher ϕ and lower Ω of DSM Section 1.2.2 may therefore be used.

LRFD: $\phi_b := 0.9$ $\quad \phi_b \cdot M_n = 84\,kip \cdot in$

ASD: $\Omega_b := 1.67$ $\quad \frac{M_n}{\Omega_b} = 56\,kip \cdot in$

Design examples provided in Chapter 8.

Symbols and definitions

Unless explicitly defined herein, variables referred to in this Guide are defined in the *Specification* (AISI 2001, 2004) or the *Design Manual* (AISI 2002).

An abbreviated list of variables is provided here for the reader's convenience.

$M_{cr\ell}$ Critical elastic local buckling moment determined in accordance with Appendix 1 (DSM) Section 1.1.2
M_{crd} Critical elastic distortional buckling moment determined in accordance with Appendix 1 (DSM) Section 1.1.2
M_{cre} Critical elastic lateral-torsional bucking moment determined in accordance with Appendix 1 (DSM) Section 1.1.2
$M_{n\ell}$ Nominal flexural strength for local buckling determined in accordance with Appendix 1 (DSM) Section 1.2.2.2
M_{nd} Nominal flexural strength for distortional buckling determined in accordance with Appendix 1 (DSM) Section 1.2.2.3
M_{ne} Nominal flexural strength for lateral-torsional buckling determined in accordance with Appendix 1 (DSM) Section 1.2.2.1
M_n Nominal flexural strength, M_n, is the minimum of M_{ne}, $M_{n\ell}$ and M_{nd}
M_y Yield moment (S_gF_y)

$P_{cr\ell}$ Critical elastic local column buckling load determined in accordance with Appendix 1 (DSM) Section 1.1.2
P_{crd} Critical elastic distortional column buckling load determined in accordance with Appendix 1 (DSM) Section 1.1.2
P_{cre} Minimum of the critical elastic column buckling load in flexural, torsional, or torsional-flexural buckling determined in accordance with Appendix 1 (DSM) Section 1.1.2
$P_{n\ell}$ Nominal axial strength for local buckling determined in accordance with Appendix 1 (DSM) Section 1.2.1.2
P_{nd} Nominal axial strength for distortional buckling determined in accordance with Appendix 1 (DSM) Section 1.2.1.3
P_{ne} Nominal axial strength for flexural, torsional, or torsional- flexural buckling determined in accordance with Appendix 1 (DSM) Section 1.2.1.1
P_n Nominal axial strength, P_n, is the minimum of P_{ne}, $P_{n\ell}$ and P_{nd}
P_y Squash load (A_gF_y)

Terms

Unless explicitly defined herein, terms referred to in this Guide are defined in the *Specification* (AISI 2001, 2004). An abbreviated list of terms is provided here for the reader's convenience.

Elastic buckling value. The load (or moment) at which the equilibrium of the member is neutral between two alternative states: the buckled shape and the original deformed shape.

Local buckling. Buckling that involves significant distortion of the cross-section, but this distortion includes only rotation, not translation, at the internal fold lines (e.g., the corners) of a member. The half-wavelength of the local buckling mode should be less than or equal to the largest dimension of the member under compressive stress.

Distortional buckling. Buckling that involves significant distortion of the cross-section, but this distortion includes rotation and translation at one or more internal fold lines of a member. The half-wavelength is load and geometry dependent, and falls between local and global buckling.

Global buckling. Buckling that does not involve distortion of the cross-section, instead translation (flexure) and/or rotation (torsion) of the entire cross-section occurs. Global, or "Euler" buckling modes: flexural, torsional, torsional-flexural for columns, lateral-torsional for beams, occur as the minimum mode at long half-wavelengths.

Fully braced. A cross-section that is braced such that global buckling is restrained.

Related Definitions from the *North American Specification for the Design of Cold-Formed Steel Structural Members* (AISI 2001)

Local Buckling. Buckling of elements only within a section, where the line junctions between elements remain straight and angles between elements do not change.

Distortional Buckling. A mode of buckling involving change in cross-sectional shape, excluding local buckling.

Torsional-Flexural Buckling. Buckling mode in which compression members bend and twist simultaneously without change in cross-sectional shape.

Allowable Design Strength. Allowable strength, R_n/Ω, (force, moment, as appropriate), provided by the structural component.

Design Strength. Factored resistance, ϕR_n (force, moment, as appropriate), provided by the structural component.

Nominal Strength. The capacity $\{R_n\}$ of a structure or component to resist effects of loads, as determined in accordance with this *Specification* using specified material strengths and dimensions.

Required Allowable Strength. Load effect (force, moment, as appropriate) acting on the structural component determined by structural analysis from the nominal loads for ASD (using all appropriate load combinations).

Required Strength. Load effect (force, moment, as appropriate) acting on the structural component determined by structural analysis from the factored loads for LRFD (using all appropriate load combinations).

Table of Contents

1 Introduction

The purpose of this Guide is to provide engineers with practical guidance on the use of the Direct Strength Method (DSM) for the design of cold-formed steel members. The Direct Strength Method was adopted as Appendix 1 in the *Supplement 2004 to the North American Specification for the Design of Cold-Formed Steel Structural Members* (AISI 2004). The Direct Strength Method is an alternative procedure from the main *Specification* and does not rely on effective width, nor require iteration, for the determination of member design strength.

1.1 Using this Design Guide

The *North American Specification for the Design of Cold-Formed Steel Structural Members* (AISI 2001)
This document, referred to as the *Specification*, or main *Specification*, forms the basis for design of cold-formed steel. The Direct Strength Method, which was added to the *Specification* in 2004 as Appendix 1 provides alternative procedures to Chapters A through G, and Appendices A through C. Equation numbers in the example problems (e.g., in Chapter 8) refer to the *Specification*.

Supplement 2004 to the North American Specification for the Design of Cold-Formed Steel Structural Members (AISI 2004)
This document is a supplement to the *Specification*. Part of this supplement includes Appendix 1, Design of Cold-Formed Steel Structural Members Using Direct Strength Method, which is the subject of this Guide. The commentary of Appendix 1 is particularly important for understanding the background of the Direct Strength Method. *For use of this Guide Appendix 1 of the Supplement is needed*.

AISI *Cold-Formed Steel Design Manual* (AISI 2002)
This *Design Manual* is not required for using this Guide. However, many of the design examples presented here are based directly on the examples presented in the *Design Manual*. The member cross-section designation provided in the *Design Manual* is used in the design examples of this Guide. In addition, much of the commentary comparing the Direct Strength Method to the *Specification* is derived from the examples in the *Design Manual*.

CUFSM (Schafer 2005) Finite Strip Software
This freely available open source software, CUFSM, is utilized extensively in this Guide for elastic buckling determination of cold-formed steel members (www.ce.jhu.edu/bschafer/cufsm). However, CUFSM is not required to utilize this Guide, as (1) closed-formed solutions are provided for standard shapes, and (2) other software including CFS (www.rsgsoftware.com) and THIN-WALL (www.civil.usyd.edu.au/case/thinwall.php) are available.

1.2 Why use DSM (Appendix 1) instead of the main *Specification*?

The design of optimized cold-formed steel shapes is often completed more easily with the Direct Strength Method than with the main *Specification*. As Figure 1 indicates, DSM provides a design method for complex shapes that requires no more effort than for normal shapes, while the main *Specification* can be difficult, or even worse, simply inapplicable. Practical advantages of DSM:

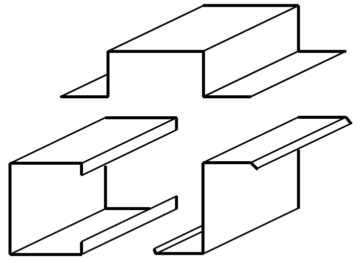
- no effective width calculations,
- no iterations required, and
- uses gross cross-sectional properties.

Elastic buckling analysis performed on the computer (e.g., by CUFSM) is directly integrated into DSM. This provides a general method of designing cold-formed steel members and creates the potential for much broader extensions than the traditional *Specification* methods, that rely on closed-form solutions with limited applicability. Theoretical advantages of the DSM approach:

- explicit design method for distortional buckling,
- includes interaction of elements (i.e., equilibrium and compatibility between the flange and web is maintained in the elastic buckling prediction), and
- explores and includes all stability limit states.

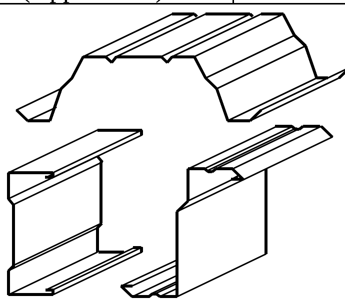
Philosophical advantages to the DSM approach:

- encourages cross-section optimization,
- provides a solid basis for rational analysis extensions,
- potential for much wider applicability and scope, and
- engineering focus is on correct determination of elastic buckling behavior, instead of on correct determination of empirical effective widths.



(a) conventional shapes

	design effort
main *Specification*	medium
DSM (Appendix 1)	medium



(b) optimized shapes

	design effort
main *Specification*	high or NA*
DSM (Appendix 1)	medium

*NA = not applicable or no design rules

Figure 1 Cold-formed steel shapes

Of course, numerous limitations of DSM (as implemented in AISI 2004) exist as well, not the least of which is that the method has only been formally developed for the determination of axial (P_n) and bending (M_n) strengths to date. A detailed list of limitations is presented and discussed in Section 1.4 of this Guide. Ongoing research and development is endeavoring to address and eliminate current limitations.

1.3 Designing with DSM (Appendix 1) and the main *Specification*

The Direct Strength Method is part of the *Specification*, and was formally adopted as Appendix 1 (AISI 2004). The term "main *Specification*" refers to the *Specification* Chapters A through G and Appendices A through C (excluding Appendix 1). The Direct Strength Method provides alternative predictions for M_n and P_n that may be used in lieu of equations in the main *Specification*; see Section 1.3.1 and the examples of Chapter 8 in this Guide. When using Appendix 1 in conjunction with the main *Specification* reliability is maintained by the use of the ϕ and Ω factors given in Appendix 1 as discussed in Section 1.3.2 of this Guide. In some situations the Direct Strength Method may form the basis for a rational analysis extension to the *Specification* as discussed in *Specification* A1.1(b) and detailed in Section 1.3.3 of this Guide.

1.3.1 Approved usage, M_n and P_n

DSM (Appendix 1 of AISI 2004) provides a strength prediction for M_n and P_n. These nominal flexural (beam) and axial (column) strengths are used in numerous sections of the main *Specification*. Table 1 below provides a roadmap for the replacement of M_n and P_n in the main *Specification* with the predicted values from Appendix 1. This table does not cover the extended use of DSM as a rational analysis tool, see Section 1.3.3 of this Guide.

Table 1 DSM alternative to main *Specification* calculations

DSM calculation →	Provides an alternative to the main *Specification*
M_n of 1.2.2 in Appendix 1 →	C3.1 Flexural Members – Bending • C3.1.1(a): DSM M_n is an alternative to M_n of C3.1.1(a) Nominal Section Strength [Resistance] *Procedure I* • C3.1.2: DSM M_n is an alternative to M_n of C3.1.2 Lateral-Torsional Buckling Strength [Resistance]
M_n with M_{ne}=M_y* of 1.2.2 in Appendix 1 →	• C3.1.3: DSM M_n is an alternative to S_eF_y of C3.1.3 Beams Having One Flange Through-Fastened to Deck or Sheathing (R must still be determined from C3.1.3) • C3.1.4: DSM M_n is an alternative to S_eF_y of C3.1.4 Beams Having One Flange Fastened to a Standing Seam Roof System (R must still be determined from C3.1.4)
M_n of 1.2.2 in Appendix 1 →	C3.3, C3.5, C5.1, C5.2 Combined Bending (interaction equations) • C3.3: DSM M_n is an alternative to M_n of C3.3 Combined Bending and Shear • C5.1: DSM M_n about x and y axes are alternatives to M_{nx} and M_{ny} of C5.1 Combined Tensile Axial Load and Bending • C5.2: DSM M_n about x and y axes are alternatives to M_{nx} and M_{ny} of C5.2 Combined Compressive Axial Load and Bending
M_n with M_{ne}=M_y* of 1.2.2 in Appendix 1 →	• C3.3: DSM M_n is an alternative to M_{nxo} of C3.3 Combined Bending and Shear • C3.5: DSM M_n is an alternative to M_{nxo} of C3.5 Combined Bending and Web Crippling, DSM $2M_n$ is an alternative to M_{no} of C3.5.1(c) and C3.5.2(c)
P_n of 1.2.1 in Appendix 1 →	C4 Concentrically Loaded Compression Members • C4: DSM P_n is an alternative to P_n of C4 Concentrically Loaded Compression Members. • C4.1 – C4.4: DSM F_{cre} is an alternative definition of F_e of sections C4.1 – C4.4, CUFSM is used as a rational analysis for F_e as discussed in C4.4 Nonsymmetric Sections
P_n with P_{ne}=P_y of 1.2.1 in Appendix 1 →	C3.6.2 Bearing Stiffeners (AISI 2004, Supplement) • C3.6.2: DSM P_n determined with P_{ne}=P_y is an alternative to A_eF_y in C3.6.2 Bearing Stiffeners in C-Section Members.
P_n of 1.2.1 in Appendix 1 →	C5.2 Combined Bending (interaction equations) • C5.2: DSM P_n is an alternative to P_n of C5.2

* In the main *Specification* to account for local buckling reductions of a fully braced beam the effective section modulus (S_e) is determined at yield (F_y) in several sections. The resulting capacity S_eF_y may be replaced by an equivalent DSM (App. 1) prediction by setting M_{ne}=M_y and then finding M_n via DSM (App. 1). Similarly for columns A_eF_y may be replaced by P_n determined with P_{ne}=P_y. See Chapter 8 for design examples.

1.3.2 Pre-qualified members

For DSM (Appendix 1 of AISI 2004) an important distinction exists as to whether or not a member geometry is “pre-qualified.” Tables 1.1.1-1 and 1.1.1-2 in Appendix 1 provide the geometric limitations that must be met for a member to be pre-qualified. Pre-qualified members may use the ϕ and Ω factors, which are given in Appendix 1.

A member which is not pre-qualified may still use the provisions of Appendix 1, but in such a case the method represents a rational analysis extension in accordance with Section A1.1(b) of the main *Specification* where ϕ=0.8 or Ω=2.0 as provided in that section. Further discussion of rational analysis extensions to DSM are explained in Section 1.3.3 of this Guide. For products, which do not meet the pre-qualified limits, Sections 7.3 and 7.4 of this Guide provide preliminary guidance on how to extend or create additional pre-qualified members; however standardized procedures have not been established at this time.

The limits on pre-qualified members represent the limits of available research and development, but do not necessarily represent limits on where optimum strength might be achieved. Some of the criteria for pre-qualified members are broader than the main *Specification* and thus represent a distinct advantage for using DSM. For example, a nominal yield stress (F_y) as high as 86 ksi is allowed in many cross-sections, web stiffeners are included, and the width/thickness limits on edge stiffened elements are higher than in the main *Specification*. In other instances the pre-qualified members are more limited than those in the main *Specification*. For example, the pre-qualified members all include shape ratios, i.e., limits on the web depth-to-flange width, these ratios generally do not exist in the main *Specification*.

Excerpt from Section A1.1(b) of the *North American Specification for the Design of Cold-Formed Steel Structural Members* (AISI 2001)

Where the composition or configuration of such components is such that calculation of the strength [resistance] and/or stiffness cannot be made in accordance with those provisions {i.e., the *Specification*}, structural performance shall be established from either of the following:

(a) Determine design strength or stiffness by tests, undertaken and evaluated in accordance with Chapter F

(b) Determine design strength [factored resistance] or stiffness by rational engineering analysis based on appropriate theory, related testing if data is available, and engineering judgment. Specifically, the design strength [factored resistance] shall be determined from the calculated nominal strength [resistance] by applying the following factors of safety or resistance factors:

Members		
USA and Mexico		Canada
Ω (ASD)	ϕ (LRFD)	ϕ (LSD)
2.00	0.80	0.75

1.3.3 Rational analysis

The development of DSM is incomplete. At this point only M_n and P_n are included. Further, many cross-sections that may be highly optimal for use in cold-formed steel structures fall outside of the scope of the main *Specification* and are not pre-qualified for DSM use. In such a situation an engineer is permitted to use rational analysis. Section A1.1(b) of the *Specification* specifically describes when rational analysis may be employed and an excerpt of this provision is provided on the previous page.

The most obvious rational analysis extension of DSM is for cross-sections that are not pre-qualified, as discussed in Section 1.3.2 of this Guide. Such cross-sections may use the ϕ and Ω factors for rational analysis and then proceed to replace M_n and P_n in the main *Specification* as discussed in Section 1.3.1 of this Guide.

A number of situations may exist where a rational analysis application of the DSM provisions is logical and worthy of pursuing. In general such an extension would include

(1) determine the elastic buckling values for $M_{cr\ell}$, M_{crd}, M_{cre}, $P_{cr\ell}$, P_{crd}, P_{cre} for the unique situation envisioned, then

(2) use the established DSM strength equations of Appendix 1 to determine M_n and P_n.

Unique modeling may be required to determine the elastic buckling values. This may include specialized CUFSM analysis, specialized analytical methods, or general purpose finite element analysis (as discussed further in Chapter 2).

Examples where a rational analysis extension to DSM of the nature described above might be considered include members: built-up from multiple cross-sections, with sheeting or sheathing on one side (flange) only, with dissimilar sheathing attached on two sides, with holes, or flanged holes, or with unique bracing (e.g., lip-to-lip braces which partially restrict distortional buckling). In addition, similar rational analysis extensions could allow an engineer to include the influence of moment gradient on all buckling modes, influence of different end conditions on all buckling modes, or influence of torsional warping stresses on all buckling modes.

In other cases rational analysis extensions to DSM may be nothing more than dealing with the situation where an observed buckling mode is difficult to identify and making a judgment as to how to categorize the mode. The basic premise of DSM: extension of elastic buckling results to ultimate strength through the use of semi-empirical strength curves, is itself a rational analysis idea. DSM provides a basic roadmap for performing rational analysis in a number of unique situations encountered in cold-formed steel design.

1.4 Limitations of DSM: practical and theoretical

Limitations of DSM (as implemented in AISI 2004)

- No shear provisions
- No web crippling provisions
- No provisions for members with holes
- Limited number/geometry of pre-qualified members
- No provisions for strength increase due to cold-work of forming

Discussion: Existing shear and web crippling provisions may be used when applicable. Otherwise, rational analysis or testing is a possible recourse. Members with holes are discussed further in Section 3.3.9 of this Guide, and this is a topic of current research. Pre-qualified members are discussed further in Section 1.3.2 and Chapter 7 of this Guide.

Practical Limitations of DSM approach

- Overly conservative if very slender elements are used
- Shift in the neutral axis is ignored
- Empirical method calibrated only to work for cross-sections previously investigated

Discussion: DSM performs an elastic buckling analysis for the entire cross-section, not for the elements in isolation. If a small portion of the cross-section (a very slender element) initiates buckling for the cross-section, DSM will predict a low strength for the entire member. The effective width approach of the main *Specification* will only predict low strength for the offending element, but allow the rest of the elements making up the cross-section to carry load. As a result DSM can be overly conservative in such cases. The addition of stiffeners in the offending element may improve the strength, and the strength prediction, significantly. Shift in the neutral axis occurs when very slender elements are in compression in a cross-section. DSM conservatively accounts for such elements as described above, as such, ignoring the small shift has proven successful. The DSM strength equations are empirical, in much the same manner as the effective width equation, or the column curves; however, the range of cross-sections investigated is quite broad. Extension to completely unique cross-sections may require consideration of new or modified DSM strength expressions.

Limitations of elastic buckling determination by finite strip method, as implemented in CUFSM

- Cross-section cannot vary along the length
    - no holes
    - no tapered members
- Loads cannot vary along the length (i.e., no moment gradient)
- Global boundary conditions at the member ends are pinned (i.e., simply-supported)
- Assignment of modes sometimes difficult, particularly for distortional buckling
- Not readily automated due to manual need to identify the modes

Discussion: Chapter 2 and Section 3.3 of this Guide discuss elastic buckling determination, and the limitations of the finite strip method (e.g., CUFSM). Guidance on more robust alternatives using general purpose finite element analysis is given in Section 2.4 of this Guide.

2 Elastic buckling: $P_{cr\ell}$, P_{crd}, P_{cre}, $M_{cr\ell}$, M_{crd}, M_{cre}

The key to the flexibility of the Direct Strength Method is that no one particular method is prescribed for determining the elastic buckling loads and/or moments: $P_{cr\ell}$, P_{crd}, P_{cre}, $M_{cr\ell}$, M_{crd}, M_{cre}. Of course, this same flexibility may lead to some complications since a prescriptive path is not provided. This chapter of the Guide complements the commentary to the Direct Strength Method which provides significant discussion regarding elastic buckling determination.

This chapter covers the definition of the basic buckling modes (Section 2.1) and provides guidance on when a mode can be ignored because it will not impact the strength prediction (Section 2.2). Four alternatives are provided and discussed for elastic buckling determination: finite strip (Section 2.3), finite element (Section 2.4), generalized beam theory (2.5), and closed-form solutions (Section 2.6).

WARNING

Users are reminded that the strength of a member is not equivalent to the elastic buckling load (or moment) of the member. The elastic buckling load can be lower than the actual strength, for slender members with considerable post-buckling reserve; or the elastic buckling load can be fictitiously high due to ignoring inelastic effects. Nonetheless, the elastic buckling load is a useful reference for determining strength via the equations of the Direct Strength Method.

2.1 Local, distortional, and global buckling

The Direct Strength Method of Appendix 1 (AISI 2004) assigns the elastic buckling behavior into three classes: local (subscript 'ℓ'), distortional (subscript 'd'), and global (subscript 'e', where the 'e' stands for Euler buckling). The DSM commentary (AISI 2004) defines elastic buckling and the three classes. The basic definitions are reviewed here for use in this Guide.

Elastic buckling value is the load (or moment) at which the equilibrium of the member is neutral between two alternative states: the buckled shape and the original deformed shape.

Local buckling involves significant distortion of the cross-section, but this distortion includes only rotation, not translation, at the internal fold lines (e.g., the corners) of a member. The half-wavelength of the local buckling mode should be less than or equal to the largest dimension of the member under compressive stress.

Distortional buckling involves significant distortion of the cross-section, but this distortion includes rotation and translation at one or more internal fold lines of a member. The half-wavelength is load and geometry dependent, and falls between local and global buckling.

Global buckling does not involve distortion of the cross-section, instead translation (flexure) and/or rotation (torsion) of the entire cross-section occurs. Global, or "Euler" buckling modes: flexural, torsional, torsional-flexural for columns, lateral-torsional for beams, occur as the minimum mode at long half-wavelengths.

Research to provide more mechanics-based definitions that can be automatically implemented in finite strip and finite element software are underway (Schafer and Adany 2005), but at this time the phenomenon-based definitions given above represent the best available.

2.2 Elastic buckling upperbounds

For all buckling modes: local, distortional, global, if the elastic buckling value is high enough then the cross-section will develop its full capacity (i.e., the yield moment in bending, M_y, or the squash load in compression, P_y). Using the Direct Strength predictor equations of Appendix 1 the following limits can be generated:

Beams

if $M_{cr\ell} > 1.66M_y$ then no reduction will occur due to local buckling

if $M_{crd} > 2.21M_y$ then no reduction will occur due to distortional buckling

if $M_{cre} > 2.78M_y$ then no reduction will occur due to global buckling

Columns

if $P_{cr\ell} > 1.66P_y$ then no reduction will occur due to local buckling

if $P_{crd} > 3.18P_y$ then no reduction will occur due to distortional buckling

if $P_{cre} \geq 3.97P_y$ a 10% or less reduction will occur due to global buckling
if $P_{cre} \geq 8.16P_y$ a 5% or less reduction will occur due to global buckling
if $P_{cre} \geq 41.64P_y$ a 1% or less reduction will occur due to global buckling

Notes:

- When considering the limits above for local buckling, the given values are conservative. Since local buckling interacts with global buckling, M_y and P_y can be replaced by M_{ne} and P_{ne}, for the local buckling upperbounds, where M_{ne} and P_{ne} are the nominal strengths determined in Appendix 1.
- When comparing local and distortional buckling, distortional buckling is likely to result in a lower strength at higher elastic buckling values than local buckling.
- Due to the nature of the global buckling column curve, some reduction in the strength is nearly inevitable due to global buckling.

These elastic buckling limits have a number of useful purposes.

- In optimizing a cross-section, stiffeners or other modifications that increase elastic buckling loads [or moments] higher than the limits given above will not impact the final strength.
- In performing a finite strip analysis it may be difficult to identify a particular buckling mode, this often occurs when the mode is at a relatively high load [or moment]. If the buckling load or moment of the mode is higher than the limits given above determination of its exact value is not necessary since it will not impact the final strength.
- In performing elastic buckling finite element analysis (often called a stability eigenvalue or eigenbuckling analysis) it is common to need a range over which the buckling loads [or moments] should be determined. The limits above provide a conservative approximation of this range.

2.3 Finite strip solutions

This section of the Guide discusses basic elastic buckling determination using the finite strip method. The Direct Strength Method emphasizes the use of finite strip analysis for elastic buckling determination. Finite strip analysis is a general tool that provides accurate elastic buckling solutions with a minimum of effort and time. Finite strip analysis, as implemented in conventional programs, does have limitations, the two most important ones are

- the model assumes the ends of the member are simply-supported, and
- the cross-section may not vary along its length.

These limitations preclude some analysis from readily being accomplished with the finite strip method, but despite these limitations the tool is useful, and a major advance over plate buckling solutions and plate buckling coefficients (k's) that only partially account for the important stability behavior of cold-formed steel members. Overcoming specific difficulties associated with elastic buckling determination by the finite strip method is discussed in Section 3.3 of this Guide, following the detailed examples of Chapter 3 of this Guide.

2.3.1 CUFSM and other software

The American Iron and Steel Institute has sponsored research that, in part, has led to the development of the freely available program, CUFSM, which employs the finite strip method for elastic buckling determination of any cold-formed steel cross-section. The program is available at www.ce.jhu.edu/bschafer/cufsm and runs on any PC with Windows 9x, NT, 2000, XP. Tutorials and examples are available online at the same address. The analyses performed in this Guide employ CUFSM. Users of this Guide are encouraged to download and use the software.

The basic steps for performing any finite strip analysis are

- define the cross-section geometry,
- determine the half-wavelengths to be investigated,
- define the applied (reference) stress; the results or load-factors are multipliers of this applied stress,
- perform an elastic buckling analysis, and
- examine the load-factor vs. half-wavelength curve to determine minimum load-factors for each mode shape.

Finite strip software

At least three programs are known to provide elastic buckling by the finite strip method:

- CUFSM (www.ce.jhu.edu/bschafer/cufsm)
- CFS (www.rsgsoftware.com), and
- THIN-WALL (www.civil.usyd.edu.au/case/thinwall.php).

2.3.2 Interpreting a solution

A finite strip analysis provides two results for understanding elastic buckling analysis, (1) the half-wavelength and corresponding load-factors, and (2) the cross-section mode (buckled) shapes. These two results are presented graphically in CUFSM as shown in along with discussions of the applied (reference) stress, which is what the load-factor (vertical axis of the response curve) refers to; minima, which are identified for each mode; half-wavelength, the longitudinal variation of the buckled shape; and mode shapes, the two-dimensional (2D) or cross-section variation of the buckled shape.

For the example in of a 9CS2.5x059 (discussed further in Section 3.2.1 of this Guide), the applied (reference) stress is that of bending about the major axis, and the maximum stress is set equal to the yield stress of the material (55 ksi) so the applied reference stress is itself the yield moment, M_y. (Note, this definition of the yield moment is referenced to the nodal locations of the model which occur at the centerline of the thickness, not at the extreme fiber. For design practice the stress level at centerline of the thickness should be back-calculated with the yield stress occurring at the extreme fiber.) Facilities exist in CUFSM for generating and applying this stress, or other common stress distributions as applied (reference) stresses.

The finite strip method always assumes the member buckles as a single half sine wave along the length. The length of this half sine wave is known as the half-wavelength. Finite strip analysis provides the buckling load [or moment] for all half-wavelengths selected by the user. Thus, finite strip analysis provides a means to understand all modes of buckling that might occur inside a given physical length (e.g., L=200 in. as shown in). The actual member buckling mode (buckled shape) considered by FSM is:

$$\text{Member buckling mode} = \text{2D mode shape} \cdot \sin(\pi x/\text{half-wavelength})$$

In the example, the analysis has been performed at a large number of half-wavelengths. Resolution of the half-wavelength curve is only necessary to this level of accuracy near the minima. Two minima are identified from the curve:

- local buckling with an $M_{cr\ell} = 0.67M_y$ and a half-wavelength of 5 in., and
- distortional buckling with an $M_{crd} = 0.85M_y$ and a half-wavelength of 25 in.

Lateral-torsional buckling is identified at longer half-wavelengths. The exact value of M^*_{cre} that would be relevant would depend on the physical length of the member. The elastic buckling moment M^*_{cre} obtained from FSM does not take into account the influence of moment gradient, so C_b from the main *Specification* (Eq. C3.1.2.1-10) should be used to determine the elastic buckling value used in the Direct Strength Method, i.e., $M_{cre}=C_b M^*_{cre}$.

Understanding Finite Strip Analysis Results

Applied stress on the section indicates that a moment about the major axis is applied to this section. All results are given in reference to this applied stress distribution. Any axial stresses (due to bending, axial load, warping torsional stresses, or any combination thereof) may be considered in the analysis.

Minima indicate the lowest load level at which a particular mode of buckling occurs. The lowest M_{cr}/M_y is sought for each type of buckling. An identified cross-section mode shape can repeat along the physical length of the member.

Mode shapes are shown at the identified minima and at 200 in. Identification of the mode shapes is critical to DSM, as each shape uses a different strength curve to connect the elastic buckling results shown here to the actual ultimate strength. In the section, *local* buckling only involves rotation at internal folds, *distortional* buckling involves both rotation and translation of internal fold lines, and *lateral-torsional* buckling involves "rigid-body" deformation of the cross-section without distortion.

Half-wavelength shows how a given cross-section mode shape (as shown in the figure) varies along its length.

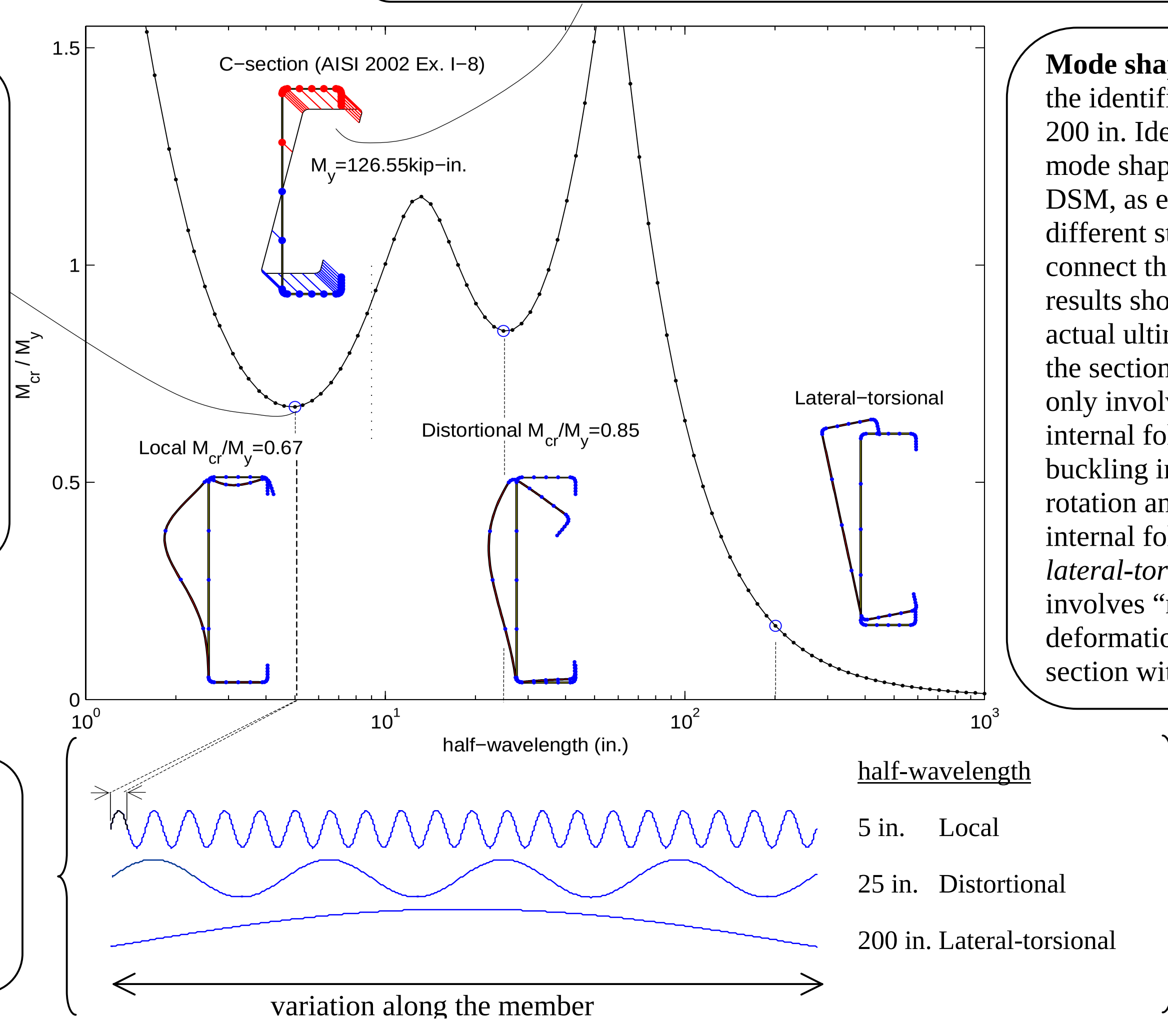


Figure 2 Understanding Finite Strip Analysis Results

2.3.3 Ensuring an accurate solution

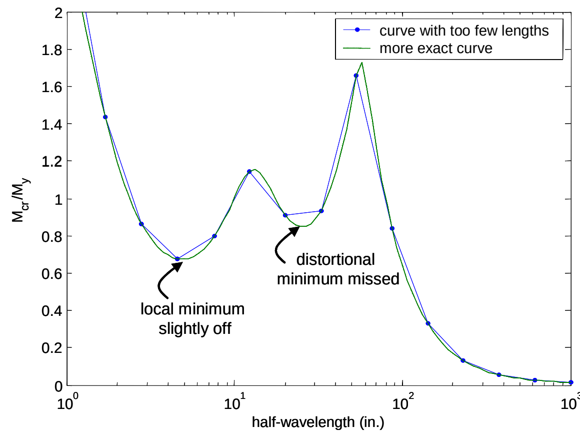


Figure 3 Importance of length selection

In a typical finite strip analysis several variables are at the user's discretion that influence the accuracy of the elastic buckling prediction, namely the number of elements in the cross-section, and the number of half-wavelengths used in the analysis.

Elements: At least two elements should always be used in any portion of a plate that is subject to compression. This minimum number of elements ensures that a local buckling wave forming in the plate will be accurate to within 0.4% of the theoretical value. If a portion of the cross-section is subject to bending it is important to ensure at least two elements are in the compression region. When examining any buckling mode shape, at least two elements should form any local buckled wave, if there is only one, increase the number of elements. The impact of the number of elements selected is greatest on local buckling and less so on distortional and global buckling.

Corners: It is recommended that when modeling smooth bends, such as at the corners of typical members, at least four elements be employed. This is a pragmatic recommendation based more on providing the correct initial geometry, rather than the impact of the corner itself on the solution accuracy. Unless the corner radius is large (e.g., r > 10t) use of centerline models that ignore the corner are adequate. The impact of corner radius is greatest on local buckling and less so on distortional and global buckling.

Half-wavelengths: As shown in Figure 3, a sufficient number of lengths should be chosen to resolve the minima points from the finite strip analysis to within acceptable accuracy. Typically the local minimum will have a half-wavelength at or near the outer dimensions of the member; however, lengths as small as any flat portion of the cross-section should be included. Distortional buckling typically occurs between three to nine times the outer dimensions of the cross-section. Global buckling is usually best examined by selecting the physical member length of interest.

2.3.4 Programming classical finite strip analysis

For readers who have programmed a conventional two dimensional matrix structural analysis code, developing a finite strip analysis code similar to CUFSM is readily doable. Cheung and Tham (1998) provide the most complete reference on the development of the finite strip method for use in solid mechanics. Schafer (1997), following Cheung's approach, provides explicit derivations of the elastic and stability matrices employed in CUFSM. CUFSM itself is open source and the routines may be easily translated from Matlab into any modern programming language. The open source code for CUFSM and the relevant Chapter from Schafer (1997) are available online at www.ce.jhu.edu/bschafer/cufsm.

2.4 Finite element solutions

This Guide cannot provide a full summary of the possible pitfalls with general purpose finite element (FE) analysis for elastic buckling (often termed eigenvalue buckling or eigen stability analysis) of cold-formed steel cross-sections. However, the use of general purpose FE analysis in the elastic buckling determination of cold-formed steel cross-sections is possible, and in some cases essentially the only known recourse. Plate or shell elements must be used to define the cross-section.

References to basic FE texts on this subject are provided in the Direct Strength Method commentary (AISI 2004). A number of commercial FE implementations exist that the author of this guide has successfully used: ABAQUS, ANSYS, MSC/NASTRAN, STAGS. In addition, ADINA, and MARC are known to reliably provide solutions for elastic buckling of plates and shells. Other programs may work equally well.

The rational analysis (see Section 1.3.3 of this Guide) extensions made possible by general purpose FE analysis include: the ability to handle any boundary condition, explicitly consider moment gradient, handle members with holes or thickness variation along the length, etc. These advantages make the use of FE analysis attractive. However, a number of theoretical and practical considerations must be handled, the most important of which are to perform benchmark problems on the elements and meshes being used, and to be patient and thorough in visually inspecting the buckling modes and identifying and assigning the buckled shapes to local, distortional, and global buckling. A discussion of some basic issues to consider when performing FE stability analysis follows.

Benchmark problems with known classical solutions (e.g., see Timoshenko and Gere 1961, Allen and Bulson 1980, Galambos 1998) should be performed with any element and mesh being considered. Section 2.6 and Chapter 9 of this Guide provide a number of potential closed-form solutions which could be used for benchmark solutions.

<u>Numerical/theoretical FE issues</u>

Element shape function: typically plate and shell elements use either linear (4 node shells) or polynomial shape functions (8 or 9 node shells) to determine how the element may deform. Regardless of the choice, a sufficient number of elements are required so that the element may adequately approximate the buckled shape of interest. A mesh convergence study with a simple benchmark problem is recommended.

Element aspect ratio: some plate and shell elements in current use will provide spurious solutions if the length/width of the element is too large or too small. Problematic element aspect ratios are element, geometry, and load dependent; however good practice is to keep elements at aspect ratios between 1:2 and 2:1, though between 1:4 and 4:1 is generally adequate.

Element choice: some plate and shell elements in current use have interpretations of the shear behavior more appropriate for moderately thick shells, as a result for thin plates common in cold-formed steel these elements may be overly soft or overly stiff in shear. This is most likely to impact distortional buckling predictions.

Practical FE issues

No half-wavelength curve: FE analysis is not performed at a variety of half-wavelengths. Varying the physical length in an FE analysis is not equivalent to varying the length in a finite strip analysis, since the longitudinal deformations possible in the FE analysis remain general (not restricted to single half sine waves). In FE analysis, all buckling modes that exist within a given physical length are examined, and the analyst must look through the modes to determine their classification: local, distortional, global. The maximum buckling value of interest is known (from the upperbounds of Section 2.2 of this Guide), but it is not known how many individual modes will be identified below this value.

Too many buckling modes are identified in a typical FE analysis for an expedient examination of the results; one must be patient and manually inspect the buckling modes. Expect to see similar buckling modes at many different half-wavelengths. It is not sufficient to identify only the minimum buckling mode. The minimum buckling value for each of the modes, local, distortional, and global needs to be identified. Engineering judgment will likely be required to identify all the modes, take care to ensure the deformed shapes are well represented by the selected element and mesh, consider performing supplementary analyses to verify results.

2.5 Generalized Beam Theory

Elastic buckling determination may also be performed using the Generalized Beam Theory (GBT). GBT references are provided in the commentary to the Direct Strength Method. Although general purpose software is not currently available in the public domain, recently Camotim and Silvestre (2004) provided GBT code for a closed-form solution for distortional buckling of C's and Z's with ends that are pinned, free, or fixed. The code may be downloaded at www.ce.jhu.edu/bschafer/gbt and provides a means to handle the influence of boundary conditions on distortional buckling of traditional cold-formed steel shapes without recourse to general purpose FE solutions.

2.6 Manual elastic buckling solutions

While the emphasis of the Direct Strength Method is on numerical solutions for elastic buckling, situations arise when manual (closed-form) solutions can be beneficial. Manual solutions may be used to provide a conservative check on more exact solutions, to readily automate elastic buckling solutions of a specific cross-section, or to help augment the identification of a particular mode in a more general numerical solution. The commentary to the Direct Strength Method provides extensive references for manual elastic buckling solutions of cold-formed steel members in local, distortional, and global buckling.

In Chapter 9 of this Guide manual elastic buckling solutions of a cold-formed C are provided for local, distortional, and global buckling of both a column and a beam.

Can elastic buckling solutions be combined? **Yes**. It is possible that the most expedient solution for a given cross-section will be to perform finite strip or finite element analysis for local and distortional buckling, but use classical formulas for global buckling; or any combination thereof.

3 Member elastic buckling examples by the finite strip method

Examples of the elastic buckling determination for members by the finite strip method, and a detailed discussion of overcoming difficulties with elastic buckling, are the focus of this chapter of the Guide. Subsequently, the cross-sections analyzed in this chapter are used extensively to examine the design of beams (Chapter 4), columns (Chapter 5), and beam-columns (Chapter 6). Complete design examples for all the cross-sections covered in this chapter are provided in Chapter 8.

3.1 Construction of finite strip models

Following the guidelines of Section 2.3.3 of this Guide a series of finite strip models were constructed. All of the models use centerline geometry for their calculation, and include corner radii. Inclusion of corner radii is not specifically necessary, but for a more exact comparison with existing models it was incorporated here.

3.2 Example cross-sections

The examples presented include those of the AISI (2002) *Design Manual* plus additional examples selected to highlight the use of the Direct Strength Method for more complicated and optimized cross-sections. For each example the following is provided: (1) references to the AISI (2002) *Design Manual* example problems (as appropriate), (2) basic cross-section information and confirmation of finite strip model geometry, and (3) elastic buckling analysis by the finite strip method (CUFSM) and notes on analysis.

Models of the following cross-sections were generated:

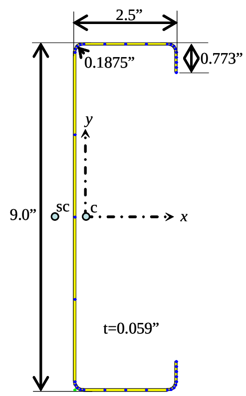
- C-section with lips,
- C-section with lips *modified*,
- C-section without lips (track section),
- C-section without lips (track section) *modified*,
- Z-section with lips,
- Z-section with lips *modified*,
- Equal leg angle with lips,
- Equal leg angle,
- Hat section,
- Wall panel section,
- Rack post section, and a
- Sigma section.

Elastic buckling results are really just another property of the cross-section

The results presented here can be thought of as augmenting the "gross properties" of the cross-section. That is, $P_{cr\ell}$, P_{crd}, $M_{cr\ell}$, M_{crd}, augment A, I, etc. as properties of the cross-section, and can be calculated without knowledge of the application of the cross-section. In the future, the elastic buckling values studied in detail in this Chapter may simply be tabled for use by engineers.

3.2.1 C-section with lips

The cross-section is a 9CS2.5x059 as illustrated in Figure 4, with F_y = 55 ksi. A model was developed in CUFSM and results are provided in Figure 5. Design examples using this cross-section are provided in Section 8.1 of this Guide. The geometry is based on Example I-1 of the AISI (2002) *Design Manual* and is also addressed in Examples I-8, II-1, and III-1 of the *Manual*.

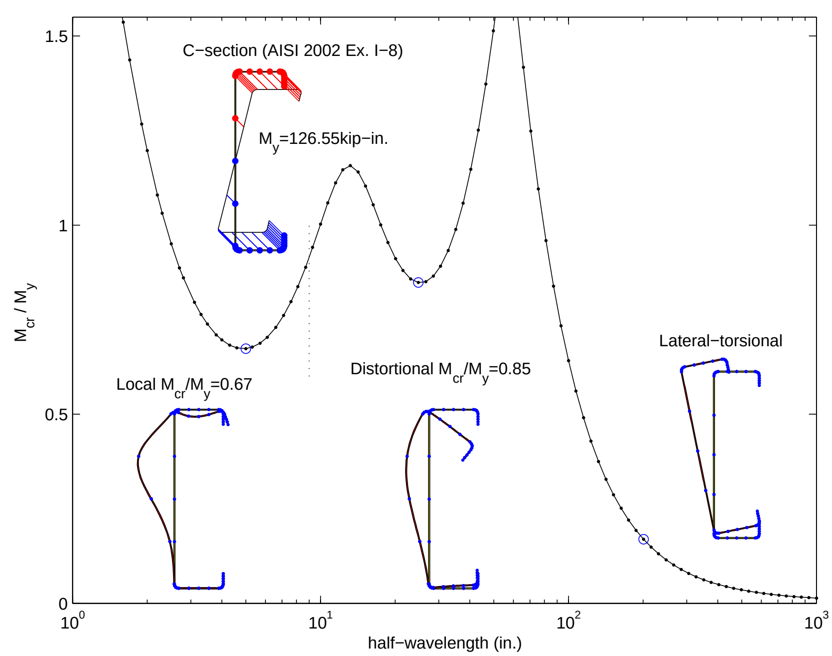


C-section with lips (9CS2.5x059)

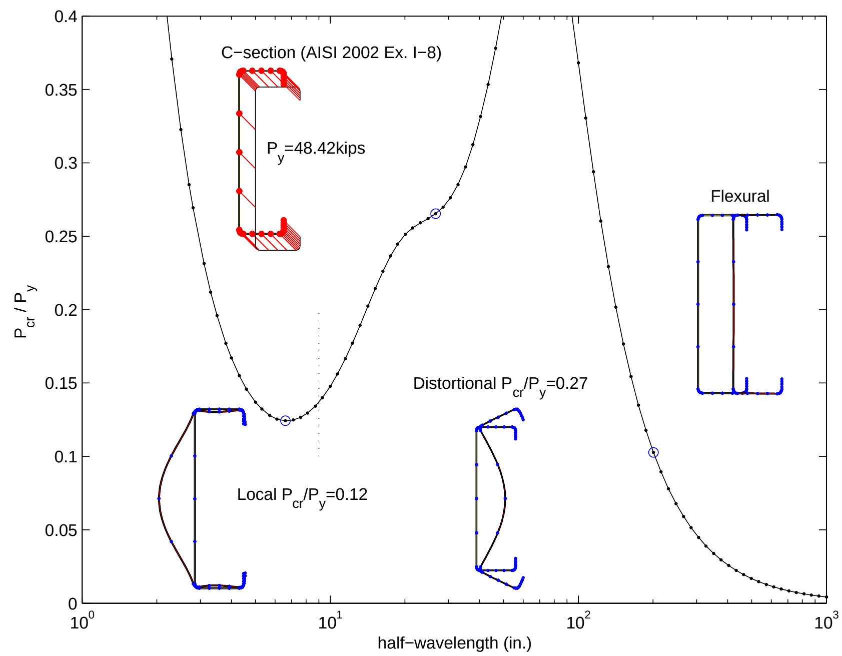
	Formula*	FSM model	
A =	0.881	0.880	in.^{2}
I_x =	10.3	10.285	in.^{4}
x_c =	0.612	0.610	in.
I_y =	0.698	0.695	in.^{4}
m =	1.048	1.036	in.
x_o =	-1.660	-1.646	in.
J =	0.00102	0.00102	in.^{4}
C_w =	11.9	11.1	in.^{6}

* given in the AISI *Design Manual* (2002)

Figure 4 C-section with lips, finite strip model and gross properties



(a) Bending



(b) Compression

Figure 5 C-section with lips, finite strip analysis results

Notes:

- The circle "○" at 200 in. in Figure 5(a) and (b) indicates the length from which the inset pictures of the global buckling mode shape are generated from. This method of indicating the length for the related global buckling mode is similar throughout this Guide. Exact values for global buckling at a given length are provided in the Design Examples of Chapter 8 as needed.
- For the analysis in pure compression (Figure 5(b)) identification of the distortional mode is not readily apparent. In this case, examination of the buckling mode shape itself (as illustrated in the Figure) identifies the transition from local to distortional buckling.
- The local buckling mode shape for pure compression shows web local buckling, but little if any local buckling in the flange and lip. When one element dominates the behavior, the strength predictions via DSM may be conservative.

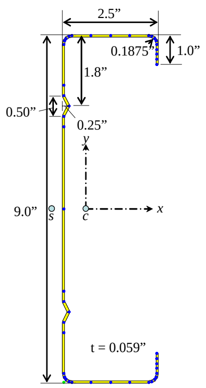
!

- M_y in this, and all, examples was generated using default options in CUFSM and thus includes the assumption that the *maximum stress occurs at the centerline of the flange* (location of the nodes in the model) instead of the extreme fiber. In this example the M_y reported above is 126.55 kip-in. and may be compared to (10.3 in.^{4}/4.5 in.)(55 ksi) = 125.89 kip-in., a difference of 0.5%. For design practice, to have the maximum stress occur at the extreme fiber, the centerline stress should be back-calculated and entered into CUFSM.

See Section 8.1 of this Guide for a complete set of design examples using this cross-section.

3.2.2 C-section with lips *modified*

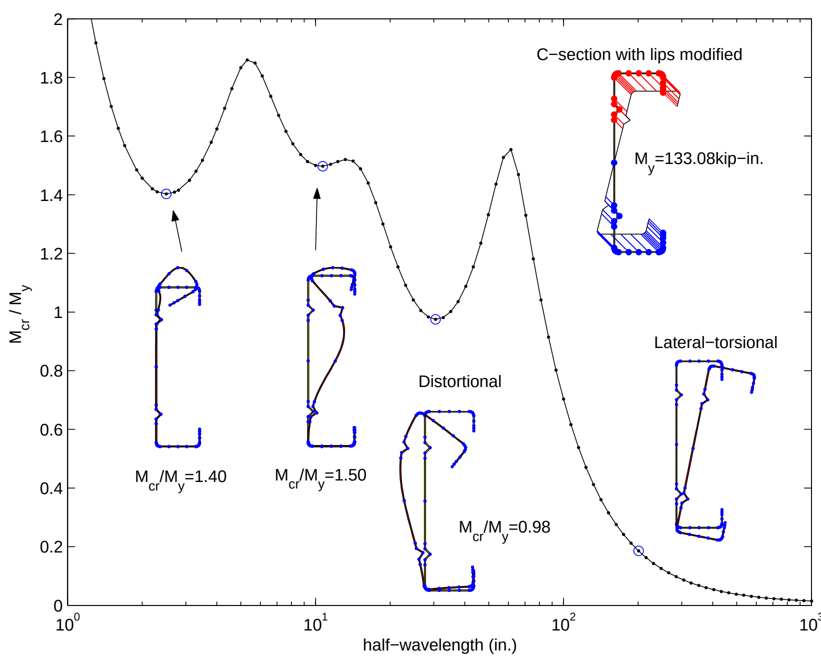
A modification of the C-section with lip (9CS2.5x059) from the previous cross-section was created to demonstrate how Appendix 1 may be applied to more unique cross-sections and to demonstrate potential preliminary steps towards cross-section optimization. The cross-section (Figure 6) has the same outer dimensions; however, 2 small ¼ in. stiffeners were added to the web, and the lip was lengthened from 0.773 in. to 1 in. Global properties are changed only slightly, so the improvement is related to local and distortional, not global, buckling. F_y remains at 55 ksi.



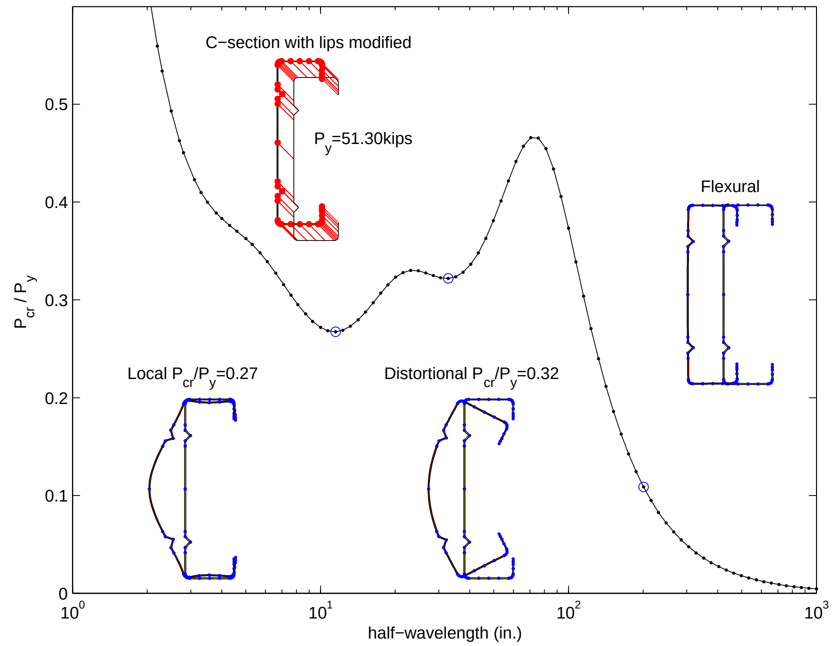
C-section with lips *modified*

	Before	After	
A =	0.880	0.933	in.^{2}
I_x =	10.285	10.818	in.^{4}
x_c =	0.610	0.659	in.
I_y =	0.695	0.781	in.^{4}
m =	1.036	1.078	in.
x_o =	-1.646	-1.859	in.
J =	0.00102	0.00108	in.^{4}
C_w =	11.1	13.33	in.^{6}

Figure 6 C-section with lips *modified,* finite strip model and gross properties



(a) Bending



(b) Compression

Figure 7 C-section with lips *modified*, finite strip analysis results

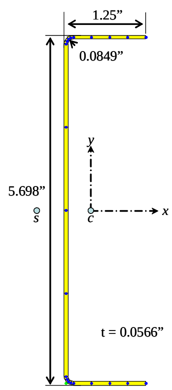
Notes:

- Compared with the conventional C-section results in Figure 5, the local buckling values increased from $0.67M_y$ to $1.40M_y$ in bending and from $0.12P_y$ to $0.27P_y$ in compression.
- Compared with the conventional C-section results in Figure 5, the distortional buckling values increased from $0.85M_y$ to $0.98M_y$ and from $0.27P_y$ to $0.32P_y$ for bending and compression, respectively.
- More complicated cross-sections can create a more complicated analysis to interpret, for example, Figure 7(a) has three minima: (1) a mode where local buckling occurs above the web stiffener, and in the flange and lip, (2) a mixed mode which is most similar to local buckling without the web stiffener, and (3) a distortional mode.

See Section 8.2 of this Guide for a complete set of design examples using this cross-section.

3.2.3 C-section without lips (track section)

The geometry for this example, a plain channel or track section, is based on Examples I-2, I-9, and II-3, of the 2002 Edition of the AISI *Cold-Formed Steel Design Manual*. The cross-section is a 550T125-54 as designated by the Steel Stud Manufacturers Association (SSMA), and is illustrated in Figure 8. Also note, F_y = 33 ksi. A model was developed in CUFSM, see Figure 9.

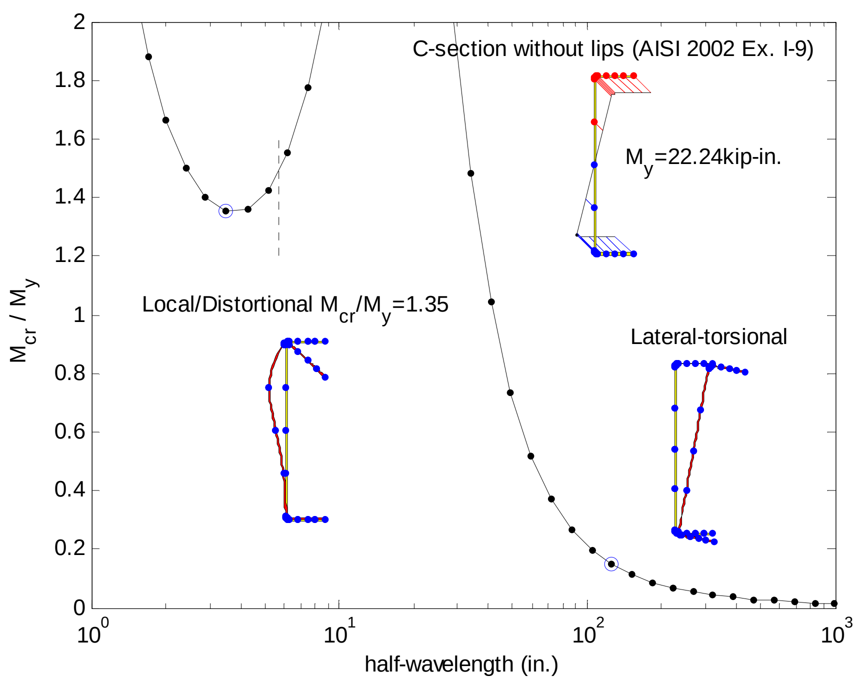


Track section (550T125-54)

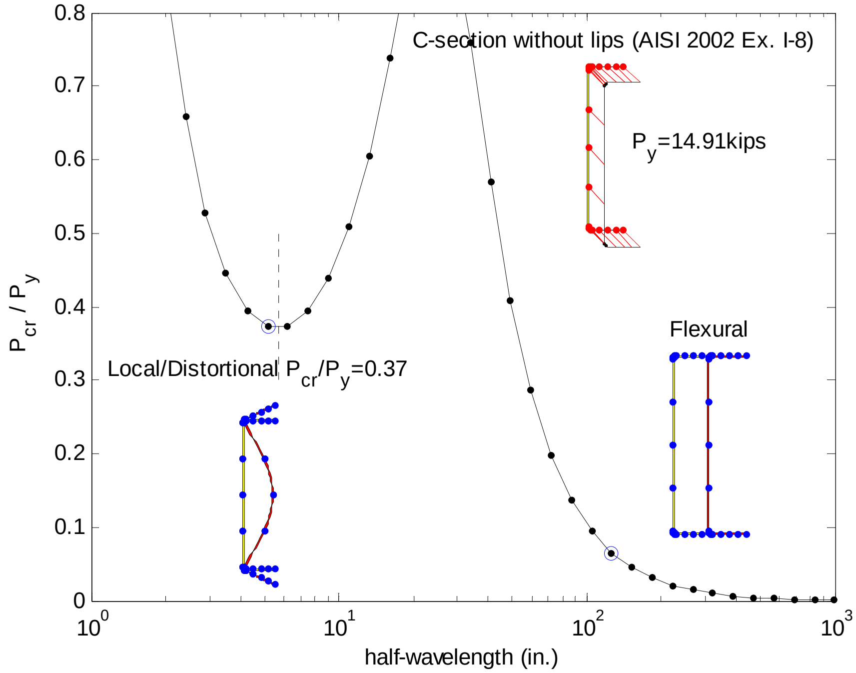
	Formula*	FSM model	
A =	0.452	0.452	in.^{2}
I_x =	1.90	1.90	in.^{4}
x_c =	0.187	0.187	in.
I_y =	0.0531	0.0528	in.^{4}
m =	0.345	0.352	in.
x_o =	-0.532	-0.538	in.
J =	0.000483	0.000483	in.^{4}
C_w =	0.316	0.307	in.^{6}

* given in the AISI *Design Manual* (2002)

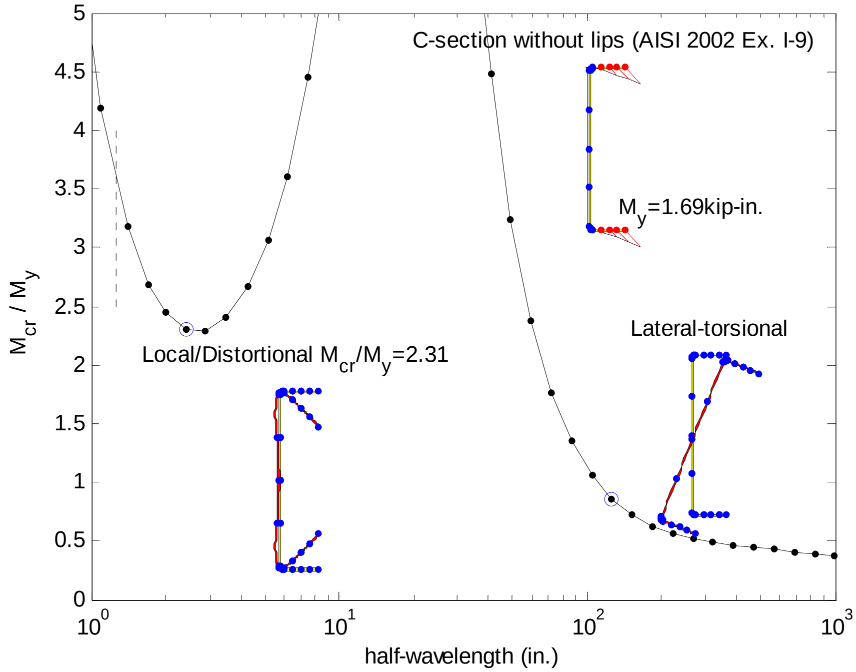
Figure 8 C-section without lips (track section), finite strip model and gross properties



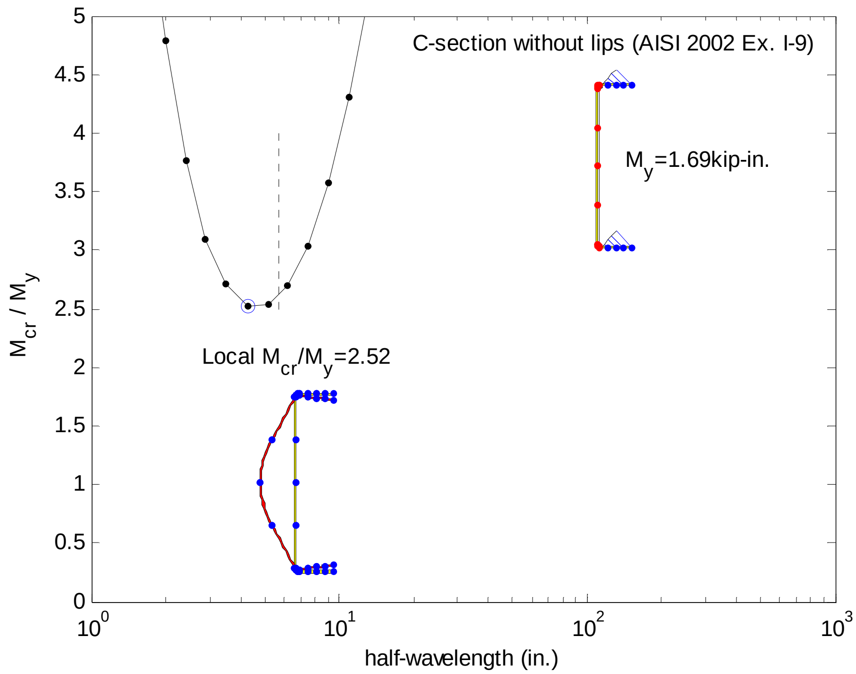
(a) Major (x-axis) bending



(b) Compression



(c) Minor (y-axis) bending
flange tips in compression



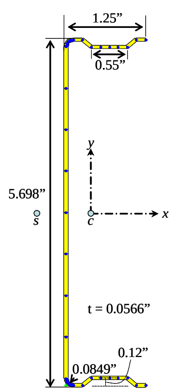
(d) Minor (y-axis) bending
flange tips in tension

Figure 9 C-section without lips (track section), finite strip analysis results

See Section 8.3 of this Guide for a complete set of design examples using this cross-section.

3.2.4 C-section without lips (track) *modified*

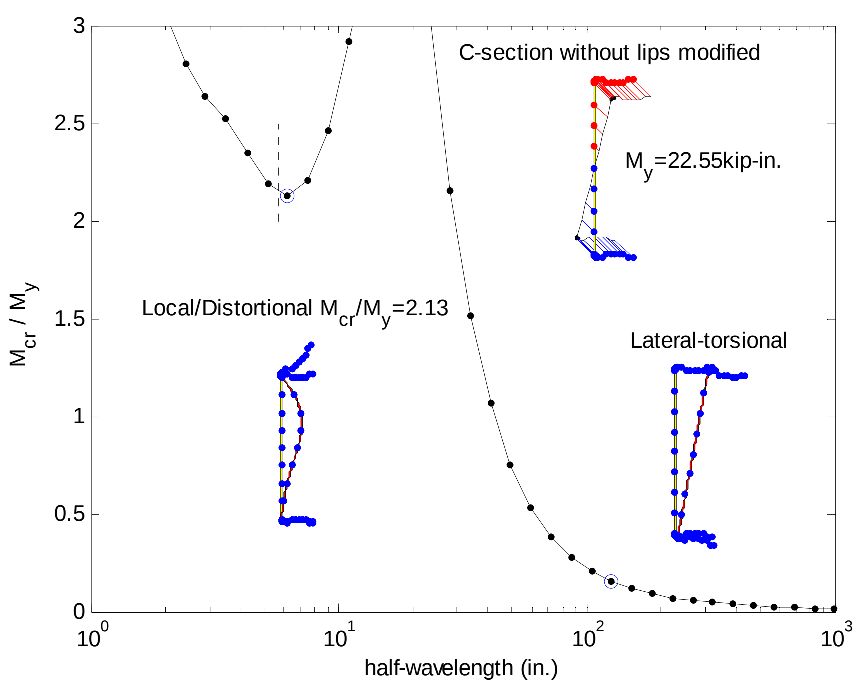
Modification of the track section of 3.2.3 is restricted by the expected use of such plain channels. If the track is envisioned as part of a conventional steel framing system, with steel studs nested in tracks, then difficulties arise with any obvious modifications: web stiffeners rolled inside the track preclude the stud from nesting fully inside the track, web stiffeners rolled outside the track preclude the track from being flush with the lower or upper floors, lip stiffeners angled inward obstruct the stud, and lip stiffeners angled outward obstruct the walls. A small corrugation on the flanges is shown as one possible alternative that would still allow the cross-section to function as a track, albeit slightly modified from conventional use. This modified SSMA 550T125-54 track section has a 0.55 in. x 0.12 in. stiffener added to the flanges as shown in Figure 10. Global properties are changed only slightly from the standard track section. F_y remains at 33 ksi.



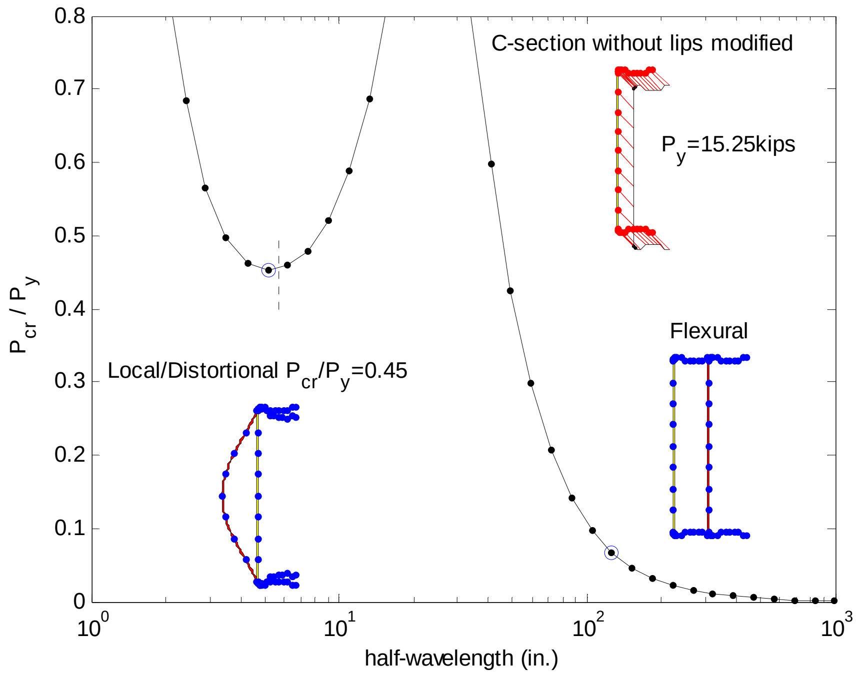
C-section without lips *modified*

	Before	After	
A =	0.452	0.463	in.^{2}
I_x =	1.90	1.93	in.^{4}
x_c =	0.187	0.198	in.
I_y =	0.0528	0.0565	in.^{4}
m =	0.35	0.36	in.
x_o =	-0.538	-0.556	in.
J =	0.00048	0.00049	in.^{4}
C_w =	0.307	0.311	in.^{6}

Figure 10 C-section without lips (track) *modified*, finite strip model and gross properties



(a) Bending



(b) Compression

Figure 11 C-section without lips (track) *modified*, finite strip analysis results

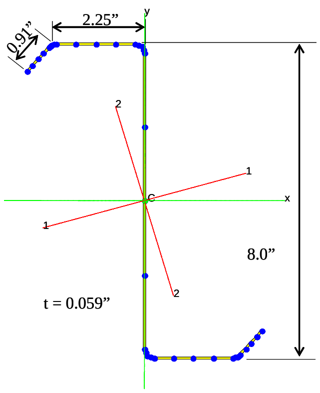
Notes:

- Weak-axis bending is not included in these calculations since the unmodified cross-section already achieved the yield capacity ($M_n = M_y$) because $M_{cr} > 2.21M_y$ (see Section 3.2.3 of this Guide)
- The first minimum in bending as well as compression is identified as local/distortional since neither the wavelength nor the mode shape itself provides a definitive separation of the mode in this case.

See Section 8.4 of this Guide for a complete set of design examples using this cross-section.

3.2.5 Z-section with lips

The geometry for this example, a Z-section, is based on Examples I-3, I-10, II-2, and III-6 of the 2002 Edition of the AISI *Cold-Formed Steel Design Manual*. The cross-section is an 8ZS2.25x059, and is illustrated in Figure 12. Also note, F_y = 55 ksi. A model was developed in CUFSM for finite strip analysis, Figure 12 illustrates the node locations of the model, a comparison of calculated cross-section properties is also provided.

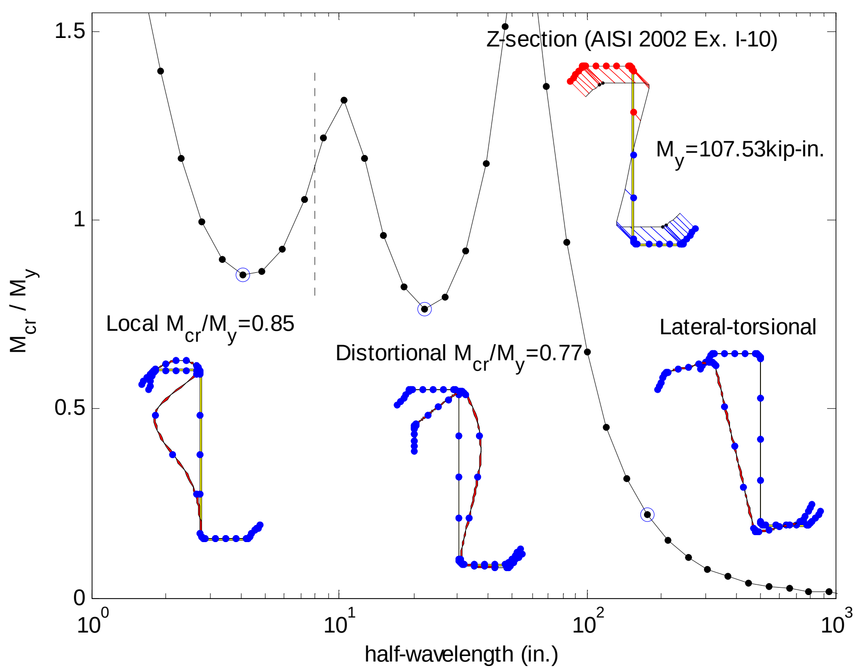


Z-section 8ZS2.25x059

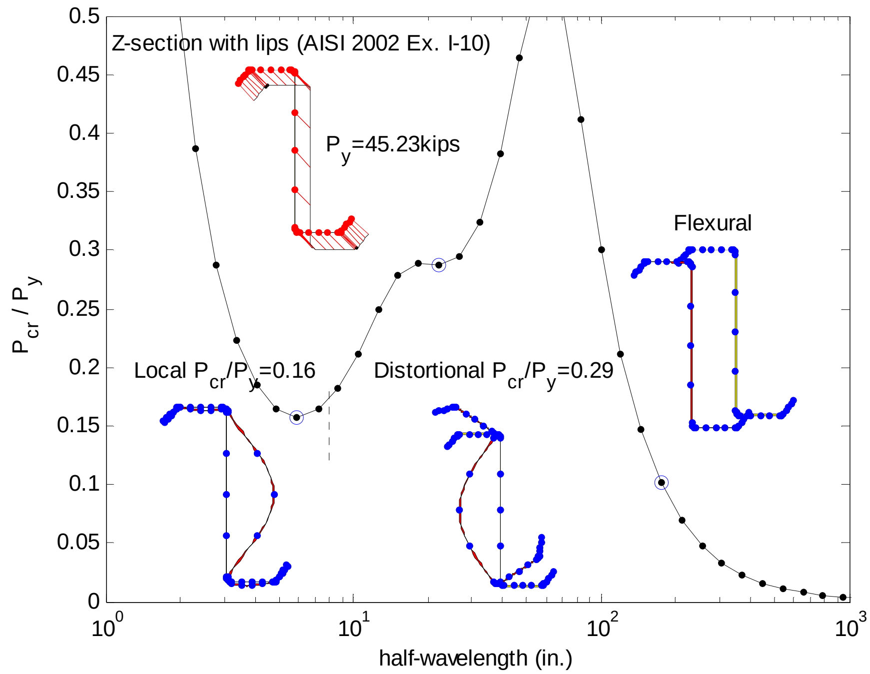
	Formula*	FSM model	
A =	0.822	0.822	in.^{2}
I_x =	7.763	7.762	in.^{4}
I_y =	1.076	1.079	in.^{4}
I_{xy} =	2.082	2.086	in.^{4}
θ =	74	74	o
I_2 =	0.481	0.481	in.^{4}
I_1 =	8.36	8.36	in.^{4}
J =	0.000954	0.000954	in.^{4}
C_w =	12.6	12.5	in.^{6}

* given in the AISI *Design Manual* (2002)

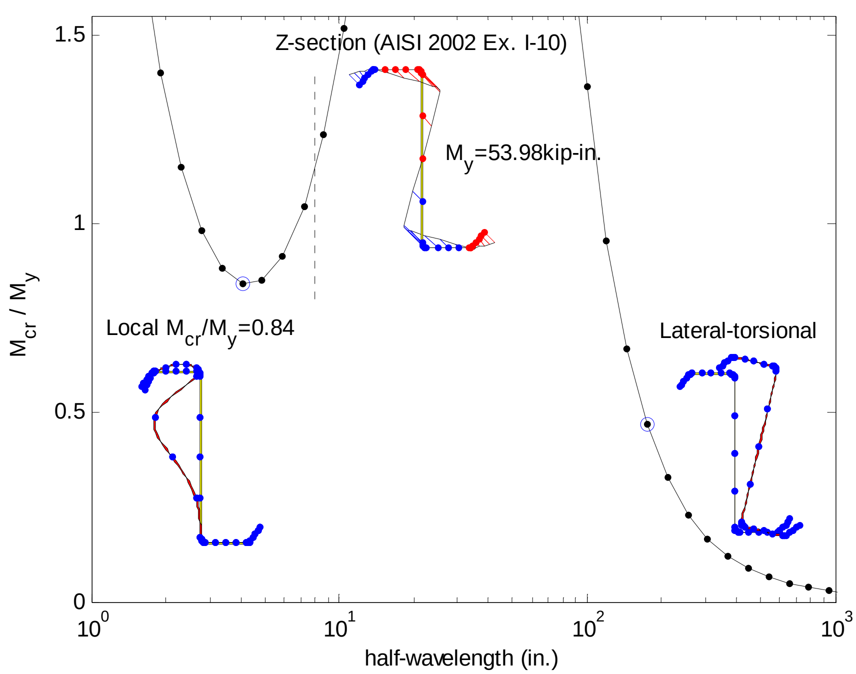
Figure 12 Z-section with lips, finite strip model and gross properties



(a) Restrained bending about x-axis



(b) Compression



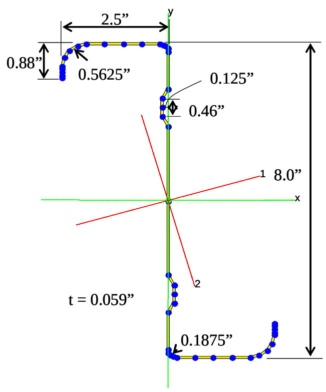
(c) Unrestrained bending about x-axis

Figure 13 Z-section with lips, finite strip analysis results

See Section 8.5 of this Guide for a complete set of design examples using this cross-section.

3.2.6 Z-section with lips *modified*

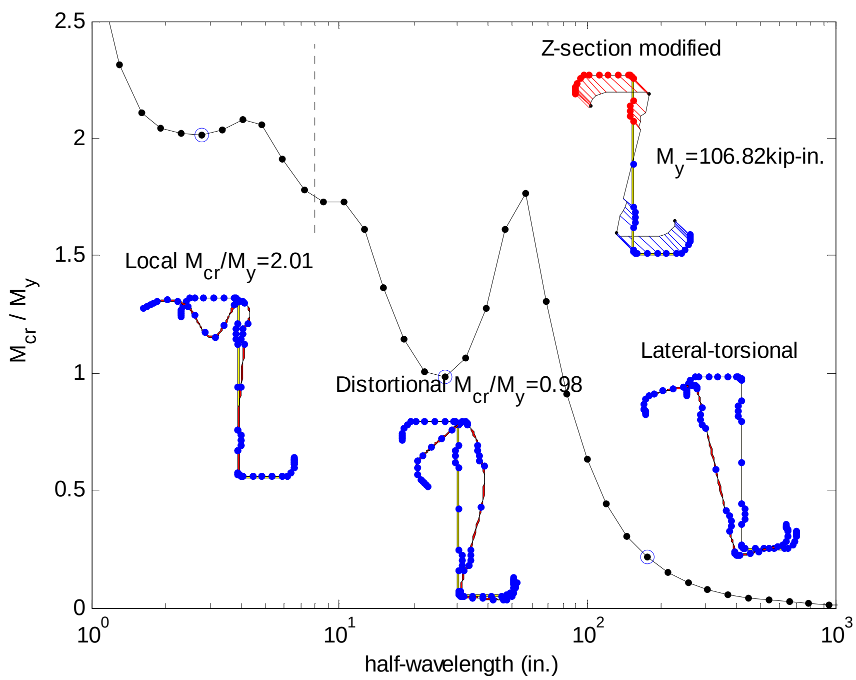
The Z-section of the previous example was modified to consider the impact of altering the geometry to achieve additional strength. The 8ZS2.25x059 was modified with two small web stiffeners and the addition of a large radius corner and lip stiffener. The resulting cross-section, illustrated in Figure 14 with F_y = 55 ksi, is not as easily "nest-able", as the traditional sloping lip Z-section, but if the flanges are made a slightly different width, nesting of the cross-sections could still be practical. A model was developed in CUFSM results are shown in Figure 15.



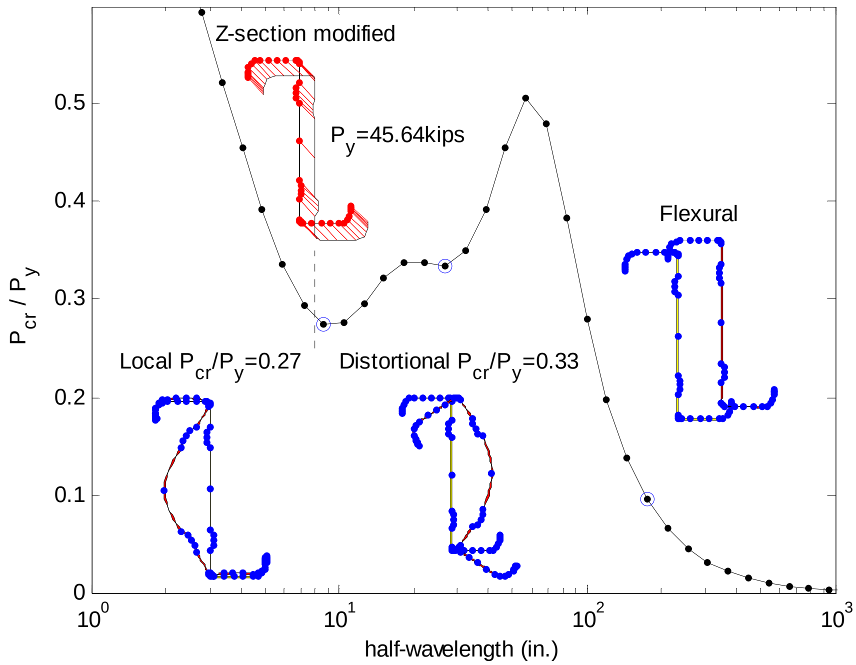
Modification of Z-section 8ZS2.25x059

	Before	After	
A =	0.822	0.830	in.^{2}
I_x =	7.762	7.653	in.^{4}
I_y =	1.079	1.044	in.^{4}
I_{xy} =	2.086	2.054	in.^{4}
θ =	74	74	^{o}
I_2 =	0.481	0.457	in.^{4}
I_1 =	8.36	8.24	in.^{4}
J =	0.000954	0.000962	in.^{4}
C_w =	12.5	11.8	in.^{6}

Figure 14 Z-section with lips *modified*, finite strip model and gross properties



(a) Restrained bending about x-axis



(b) Compression

Figure 15 Z-section with lips modified, finite strip analysis results

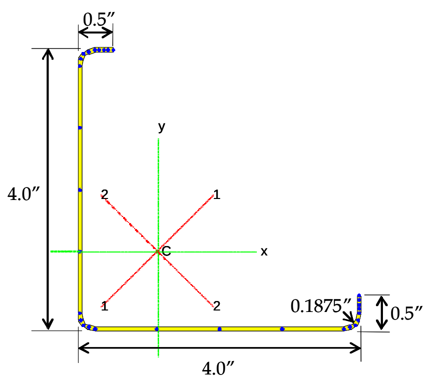
Notes:

- The improvement in local buckling (from the unmodified Z) is essentially all attributable to the addition of the web stiffeners.
- The improvement in distortional buckling (from the unmodified Z) is essentially all attributable to the changes in the lip stiffener geometry.
- Separate models with only the web stiffeners added to the original Z-section, or only the lip stiffener modified, were evaluated and support the above two notes.

See Section 8.6 of this Guide for a complete set of design examples using this cross-section.

3.2.7 Equal leg angle with lips

The geometry for an equal leg angle with lips is based on Examples I-4, I-11, and III-4 of the 2002 Edition of the AISI *Cold-Formed Steel Design Manual*. The cross-section is a 4LS4x060 as illustrated in Figure 16, with F_y = 50 ksi. Results from the CUFSM finite strip analysis are given in Figure 17.

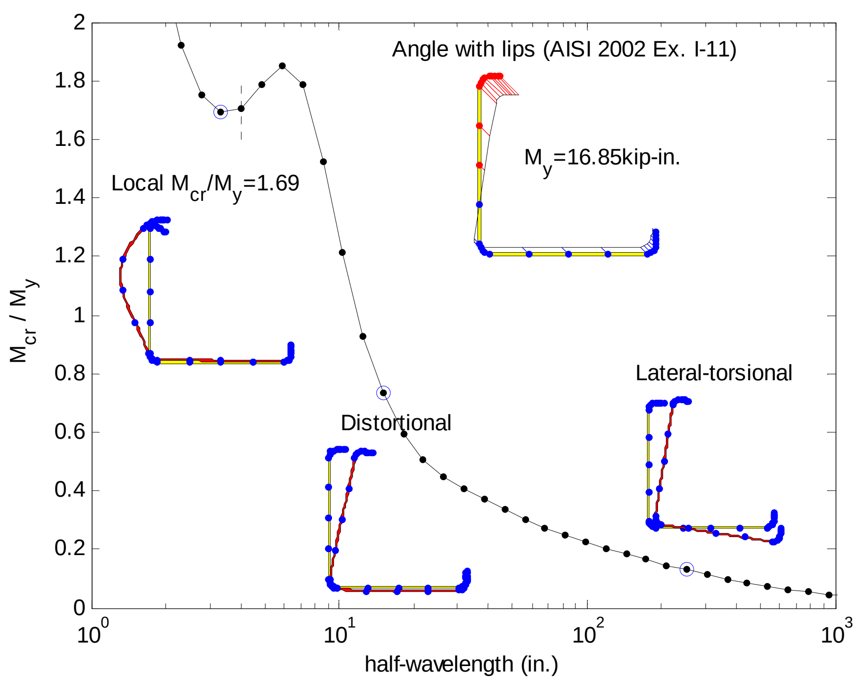


Equal leg angle with lips (4LS4x060)

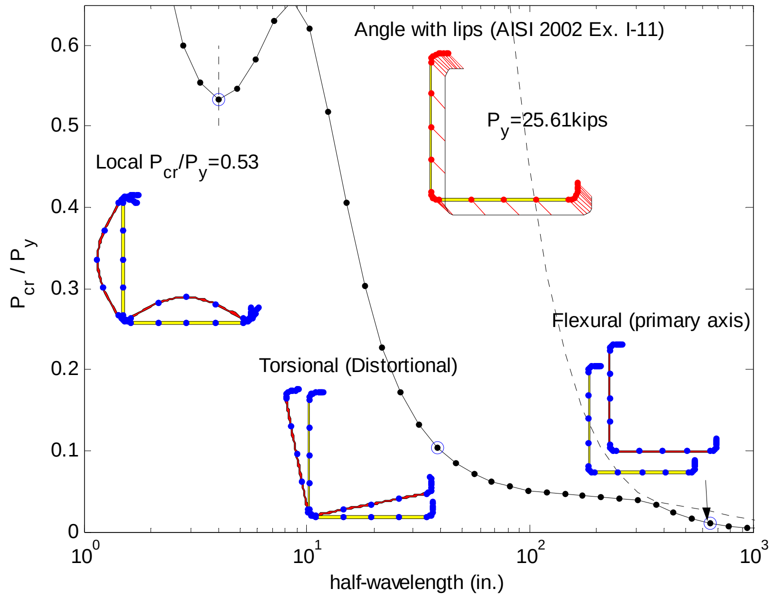
	Formula*	FSM model	
A =	0.512	0.512	in.^{2}
I_x, I_y =	0.958	0.958	in.^{4}
x_c =	1.097	1.096	in.
I_{xy} =	-0.562	-0.561	in.^{4}
I_2 =	0.396	0.397	in.^{4}
m =	0.083	0.083	in.
x_o =	-1.634	-1.633	in.
J =	0.000615	0.000615	in.^{4}

* given in the AISI *Design Manual* (2002)

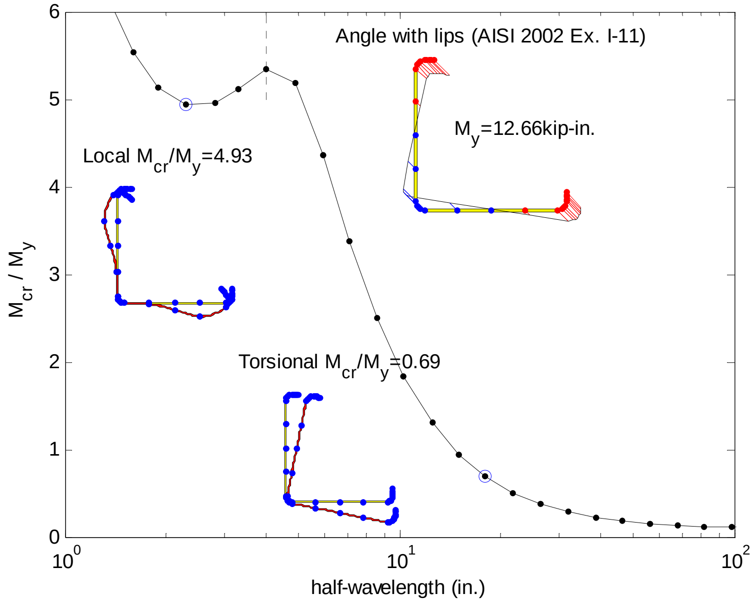
Figure 16 Equal leg angle with lips, finite strip model and gross properties



(a) Bending (geometric/restrained bending)



(b) Compression



(c) Bending about the min. principal axis(flange tips in compression)

Figure 17 Equal leg angle with lips, finite strip analysis results

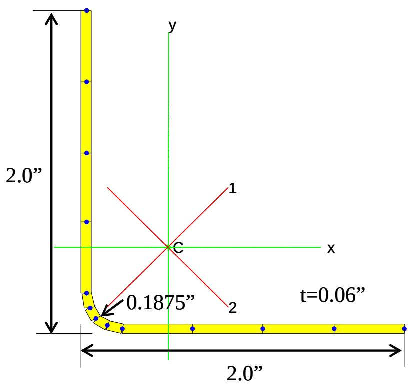
Notes:

- In bending, distortional buckling of the stiffened leg is not discretely identifiable from long-wavelength LTB (no minimum in the FSM curve). In a design the distortional buckling value (M_{cr}/M_y) at the design unbraced length should be used to determine if distortional buckling is relevant to the behavior.
- In compression, "distortional buckling" of the stiffened legs is essentially torsional buckling of the angle. The second (higher) buckling mode is presented as a dashed line in Figure 17(b). It can be seen that at long half-wavelengths flexural buckling about the minor principal axis (2-2) occurs at lower levels than torsional buckling.

See Section 8.7 of this Guide for a complete set of design examples using this cross-section.

3.2.8 Equal leg angle

The geometry selected for an equal leg angle is based on Examples I-5, and I-12 of the 2002 Edition of the AISI *Cold-Formed Steel Design Manual*. The cross-section is a 2LU2x060 as illustrated in Figure 18, with F_y = 33 ksi. A model was developed in CUFSM for finite strip analysis, results are shown in Figure 19.

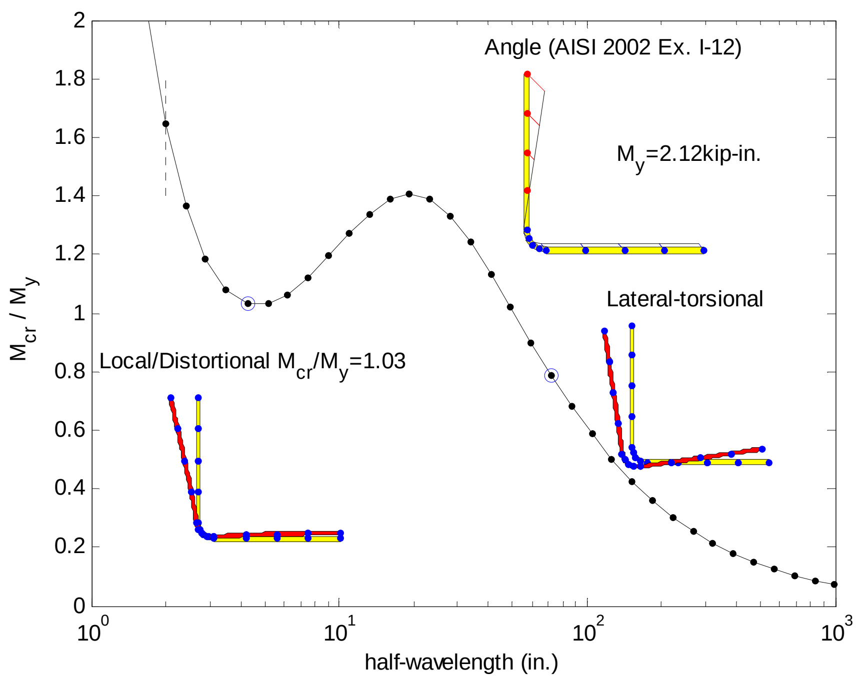


Equal leg angle (2LU2x060)

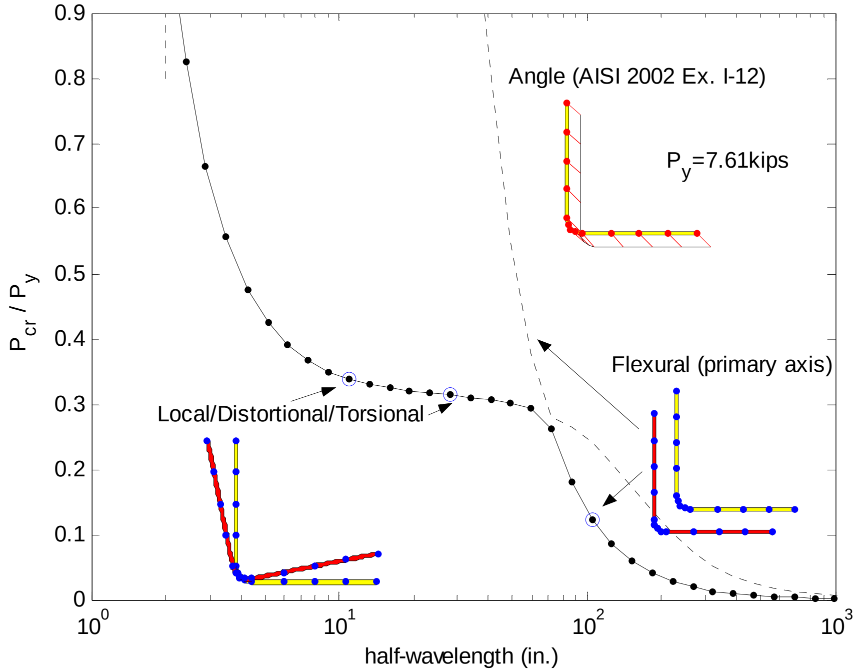
	Formula*	FSM model	
A =	0.231	0.231	in.^{2}
I_x, I_y =	0.094	0.094	in.^{4}
x_c =	0.505	0.506	in.
I_{xy} =	-0.0589	-0.0589	in.^{4}
I_2 =	0.0351	0.0350	in.^{4}
m =	0.0	0.0117	in.
x_o =	-0.714	-0.702	in.
J =	0.000277	0.000277	in.^{4}

* given in the *AISI Design Manual* (2002)

Figure 18 Equal leg angle, finite strip model and gross properties



(a) Bending (restrained bending)



(b) Compression

Figure 19 Equal leg angle, finite strip analysis results

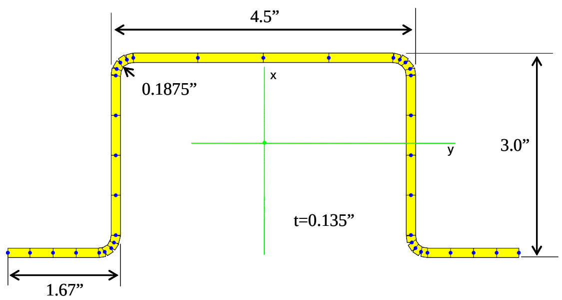
Notes:

- In bending (restrained bending about the x-axis) it is conservatively assumed that the first minimum is either local *or* distortional buckling. Based on the half-wavelength, as discussed in the DSM commentary (AISI 2004), this mode could be identified as distortional, but without an edge stiffener it has generally been considered a local mode.
- In compression, no minima exist – the analysis is always dependent on the unbraced length. At short to intermediate lengths a torsional mode dominates the buckling deformation, while at long lengths the flexural mode about the minor principal (2-2) axis dominates. For bracing to be adequate it must restrict the appropriate deformation, twist for lengths less than ~ 60 in., and bending about the minor axis for longer lengths.

See Section 8.8 of this Guide for a complete set of design examples using this cross-section.

3.2.9 Hat section

The geometry selected for a hat section is based on Examples I-6, I-13, II-4, and III-8 of the 2002 Edition of the AISI *Cold-Formed Steel Design Manual*. The cross-section is a 3HU4.5x135 as illustrated in Figure 20, with F_y = 50 ksi. A model was developed in CUFSM for finite strip analysis, with results shown in Figure 21.

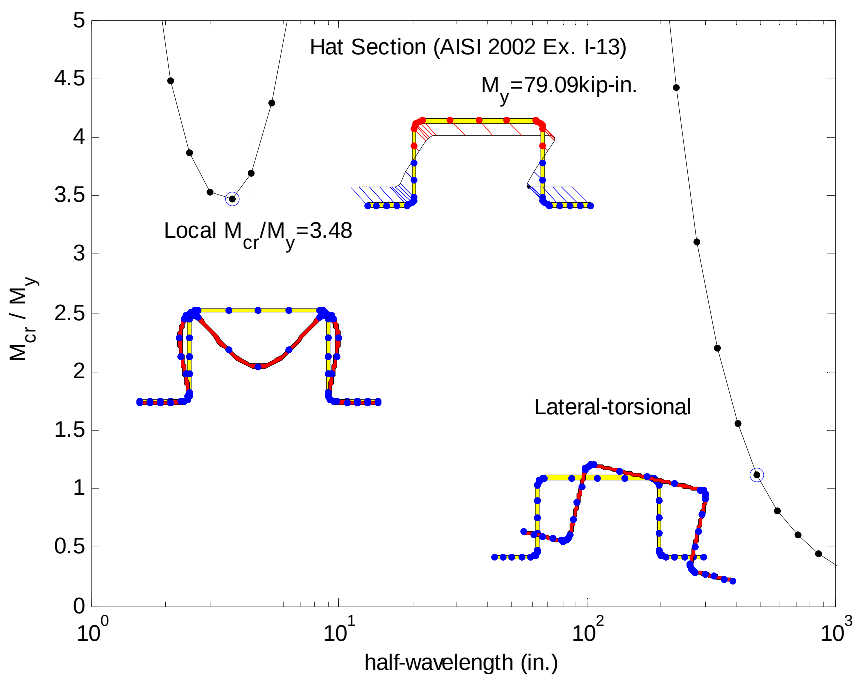


Equal leg angle (2LU2x060)

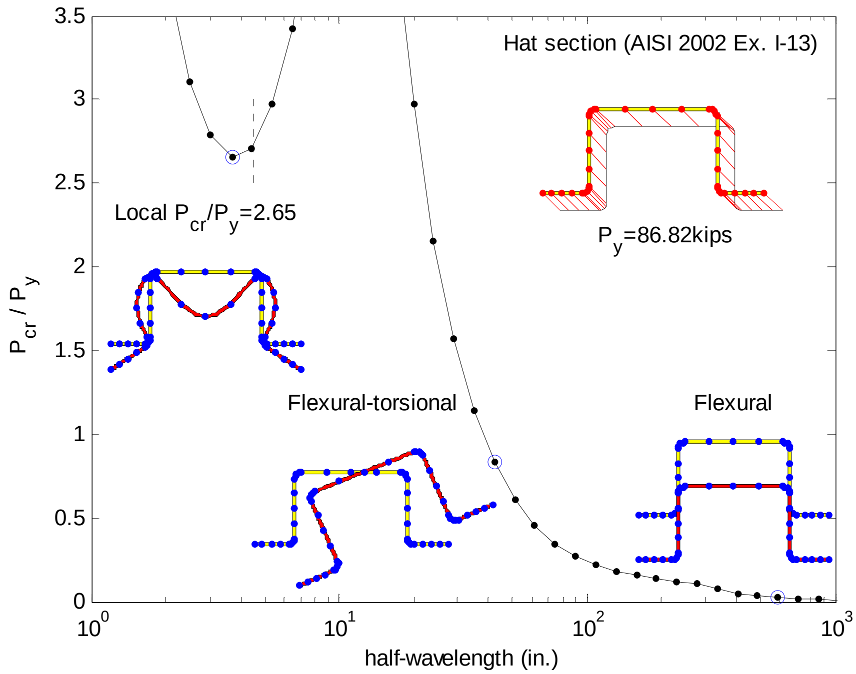
	Formula*	FSM model	
A =	1.737	1.736	in.^{2}
I_x =	8.29	8.29	in.^{4}
I_y =	2.47	2.46	in.^{4}
x_c =	1.303	1.302	in.
m =	1.182	1.16	in.
x_o =	-2.485	-2.46	in.
J =	0.0106	0.0105	in.^{4}
C_w =	5.65	4.96	in.^{6}

* given in the AISI *Design Manual* (2002)

Figure 20 Hat section, finite strip model and gross properties



(a) Bending (top flange in compression)



(b) Compression

Figure 21 Hat section, finite strip analysis results

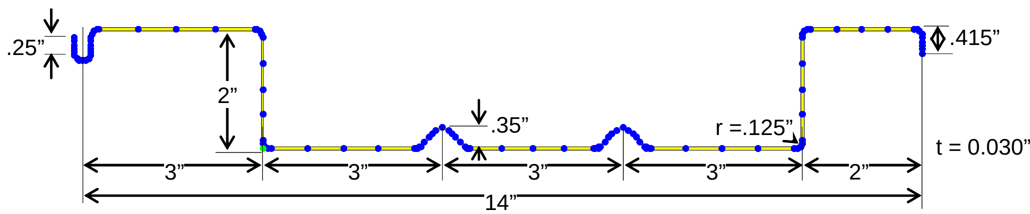
Notes:

- This hat section is relatively thick and local buckling occurs at high values (compare with the upperbounds established in Section 2.2 of this Guide), the only anticipated reductions in such a cross-section would be due to long unbraced lengths.
- Distortional buckling is not relevant to the performance of this cross-section in bending with the top flange in compression (Figure 21(a)).
- Distortional buckling occurs at too high of a stress to be relevant to the behavior of the column (Figure 21(b)).
- For long columns, the minimum mode switches from torsional-flexural to weak-axis flexural as the column length increases, at approximately 300 in. (25 ft).

See Section 8.9 of this Guide for a complete set of design examples using this cross-section.

3.2.10 Wall panel section

The geometry selected for a wall panel is based on Examples I-7, I-14, of the 2002 Edition of the AISI *Cold-Formed Steel Design Manual*. The cross-section is a 14 in. x 2 in. panel as illustrated in Figure 22, with F_y = 50 ksi. A model was developed in CUFSM for finite strip analysis (Figure 22), with results shown in Figure 23.

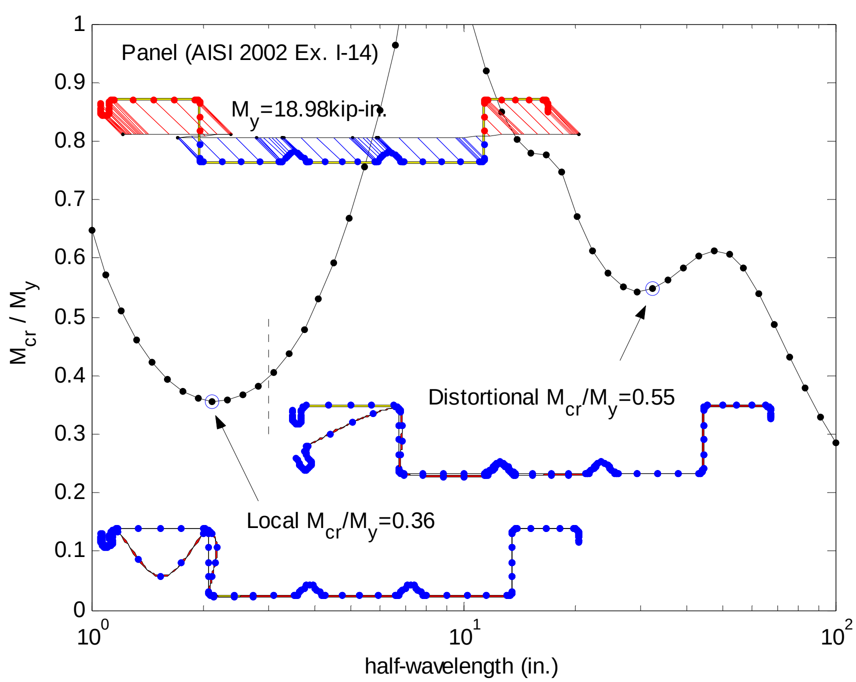


Wall Panel Section

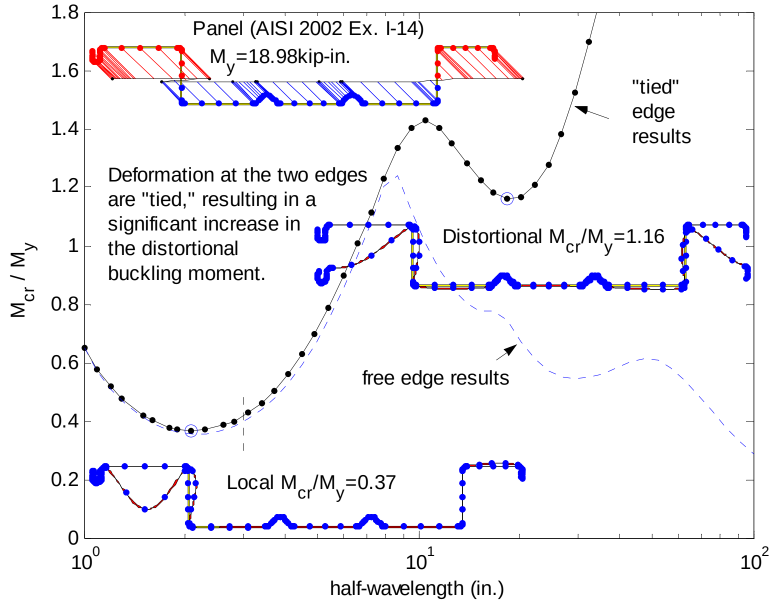
	Formula*	FSM model	
A =	0.585	0.585	in.^{2}
I_x =	0.444	0.444	in.^{4}
y_c =	1.186	1.186	in. (referenced from top fiber)

* given in the AISI *Design Manual* (2002)

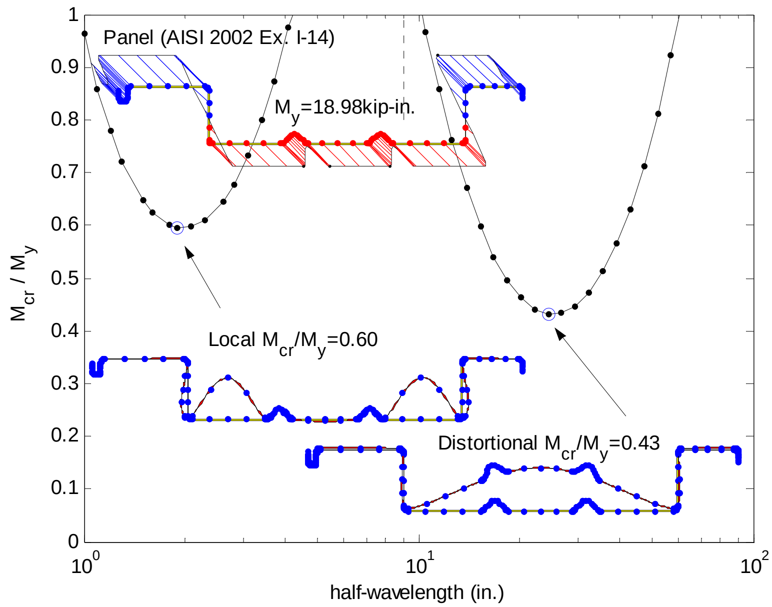
Figure 22 Wall Panel section, finite strip model and gross properties



(a) Bending (top flange in compression)
panel edges left free



(b) Bending (top flange in compression) panel edges tied to one another



(c) Bending (bottom flange in compression)

Figure 23 Wall Panel section, finite strip analysis results

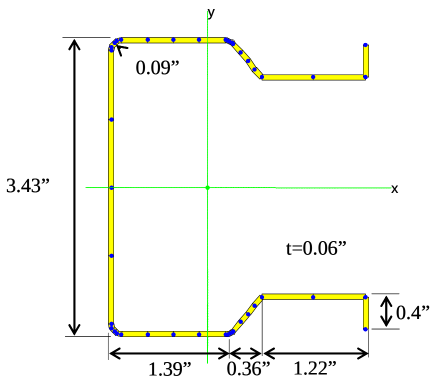
Notes:

- Boundary conditions for the edges of the panel are important when the top flanges are in compression.
- In (a) the edges are left free and distortional buckling of the panel occurs at $0.55M_y$, in (b) the edges are tied (as if by a neighboring identical panel) and the distortional buckling increases to $1.16M_y$.

See Section 8.10 of this Guide for a complete set of design examples using this cross-section.

3.2.11 Rack post section

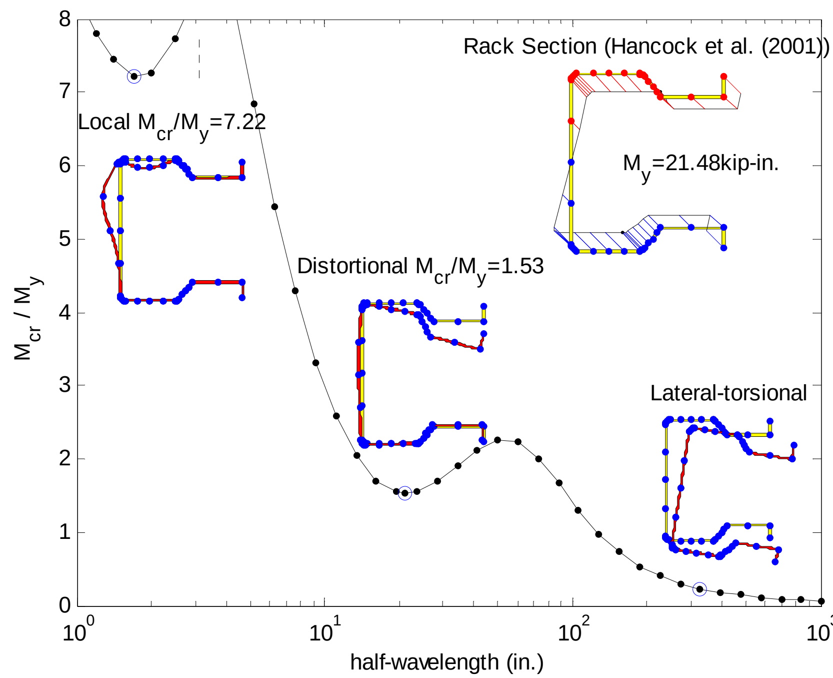
The geometry selected for a rack post section is based on the example given in Figure 3.5 of Hancock et al. (2001). The cross-section is illustrated in Figure 24, with F_y = 33 ksi. A model was developed in CUFSM for finite strip analysis, results are shown in Figure 25.



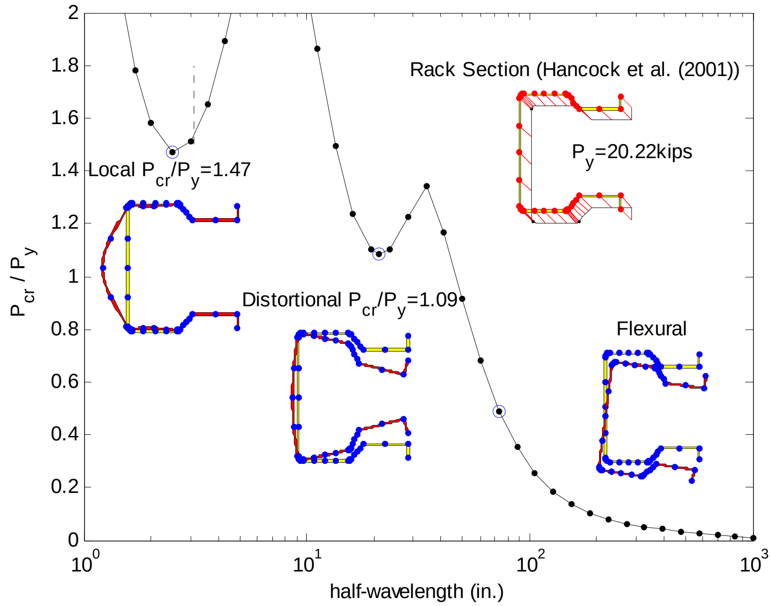
Rack Section

	FSM model	
A =	0.613	in.^{2}
I_x =	1.10	in.^{4}
I_y =	0.68	in.^{4}
x_c =	1.10	in.
m =	1.49	in.
x_o =	-2.59	in.
J =	0.000735	in.^{4}
C_w =	1.869	in.^{6}

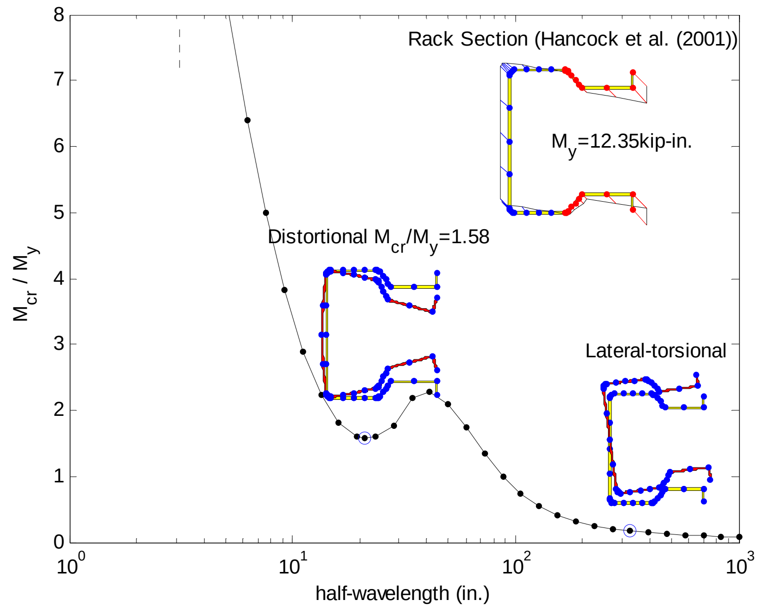
Figure 24 Rack post section, finite strip model and gross properties



(a) x-axis bending
(top flange in compression)



(b) Compression



(c) y-axis bending
(flange tips in compression)

Figure 25 Rack post section, finite strip analysis results

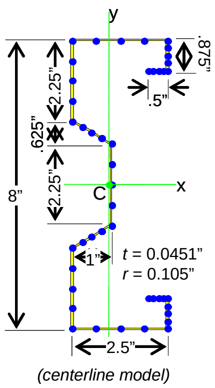
Note:

- For these cross-sections distortional buckling plays a larger role in the behavior than in many other common cross-sections.

See Section 8.11 of this Guide for a complete set of design examples using this cross-section.

3.2.12 Sigma section

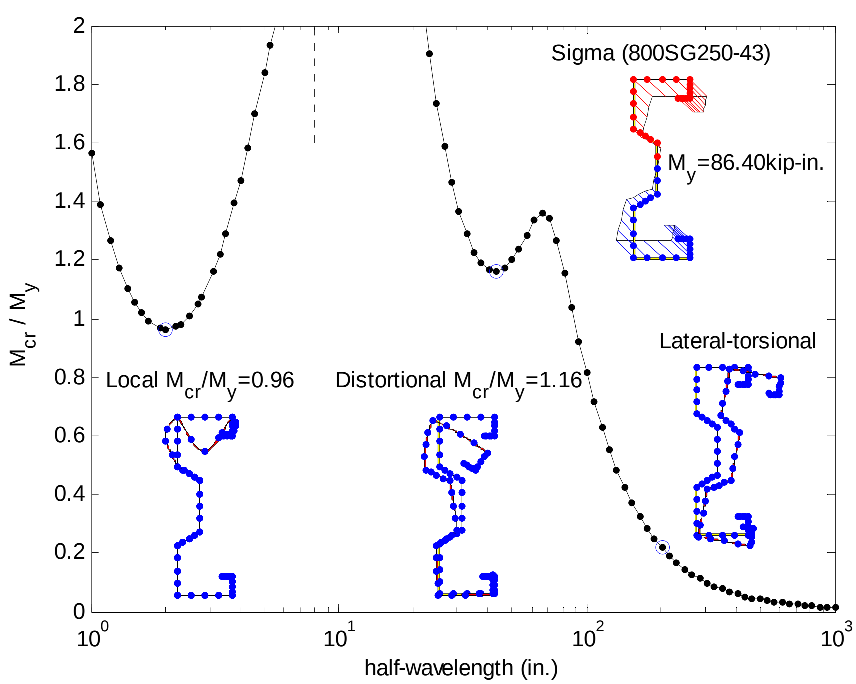
The geometry selected for a sigma section approximates a commercially available cross-section and is provided in Figure 26, where F_y = 50 ksi. No standard designation system exists for this cross-section type, but this member is designated as 800SG250-43. A model was developed in CUFSM, results are shown in Figure 27.



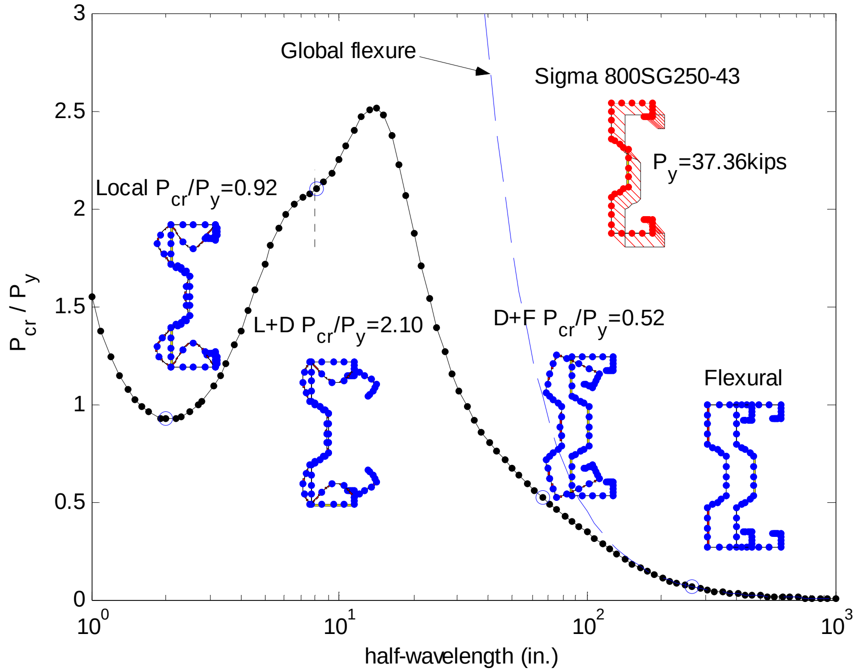
Sigma Section

	FSM model	
A =	0.747	in.^{2}
I_x =	6.878	in.^{4}
I_y =	0.576	in.^{4}
x_c =	0.940	in.
m =	0.425	in.
x_o =	-1.365	in.
J =	0.000507	in.^{4}
C_w =	11.07	in.^{6}

Figure 26 Sigma section, finite strip model and gross properties



(a) Major axis bending



(b) Compression

Figure 27 Sigma section, finite strip analysis results

Notes:

- Buckling behavior of the sigma section in bending is similar to a C-section with lips and readily useable in the Direct Strength Method, though the geometry is not pre-qualified.
- Buckling behavior of the sigma section in compression as illustrated in Figure 27 is relatively more complicated than conventional cross-sections.
  - The local buckling minimum is easily identified, but a second near minimum in the curve at a half-wavelength of approximately 8 in. includes local and distortional buckling characteristics. The short half-wavelength of this mode and its high buckling value help indicate this is not the distortional mode of interest.
  - The distortional buckling mode does not have a distinct minimum, it is dependent on the length, and interacts with the global mode (weak-axis flexure). Knowledge of the half-wavelength for distortional buckling in the bending analysis helps to identify distortional buckling in the compression analysis.

See Section 8.12 of this Guide for a complete set of design examples using this cross-section.

3.3 Overcoming difficulties with elastic buckling determination in FSM

The discussions in the following section are intended to provide the design professional with a means to apply "engineering judgment" to an elastic buckling analysis. When in doubt of how to identify a mode, or what to do with modes that seem to be interacting, or other problems; remember, it is easy to be conservative. Select the lowest buckling value (i.e., P_{cr}, M_{cr}) of all mode shapes which includes some characteristics of the mode of interest. This ensures a lower bound elastic buckling response. However, this may be too conservative in some cases, and the challenge, often, is to do better than this and use judgment to determine a more appropriate (and typically higher) approximation.

3.3.1 Indistinct local mode

It is possible that in a finite strip (or finite element) analysis that no local buckling mode is obviously identified, it is "indistinct". An indistinct local mode may occur in thicker cross-sections, where the local buckling values are quite high, or in cross-sections with very small edge stiffeners; where a distortional buckling obscures the local buckling mode.

The basic options for handling an indistinct local mode in the finite strip context include

- refine the half-wavelengths,
- review the local buckling mode definitions carefully,
- create a centerline model (no rounded corners) and pin the internal fold lines to force local buckling (see tutorials at www.ce.jhu.edu/bschafer/cufsm),
- use the manual (element) elastic buckling solutions of Section 2.6 of this Guide to provide bounds on the expected local elastic buckling values, and finally
- if all else fails one can conservatively choose the lowest buckling value which occurs in approximately the predicted half-wavelength; for local buckling this should be a length less than the largest outside dimensions of the member in compression.

Also, it is important to remember that if local bucking ($P_{cr\ell}$, $M_{cr\ell}$) is above the limits of Section 2.2 of this Guide then it may be safely ignored regardless of whether it is indistinct or not.

A special case of an indistinct local mode occurs with plain angles (Section 3.2.8, Figure 19) and channels (Section 3.2.3, Figure 9). In these cross-sections only one minimum is observed in the finite strip analysis before global buckling, but is the mode local or distortional? A key aspect of the definition of local buckling is that only rotation occurs at internal fold lines (corners) in a member. Further, local buckling should occur at a half-wavelength less than the largest member dimension under compression. The first minimum in the finite strip results meets the basic local buckling definition, but the mode shape also visually appears similar to distortional buckling for the same member with an edge stiffener added. In some cases the half-wavelength of the mode is greater than the largest member dimension under compression, in other cases it is not. The reason one identifies the elastic buckling modes in the Direct Strength Method is so that a buckling mode can be correctly associated with a given strength curve (e.g., DSM Eq. 1.2.1-6 of Appendix 1, AISI 2004). It is conservative to assume that the observed mode is both local and distortional. Alternatively, the half-wavelength of the mode could be used to place the observed mode as either local or distortional (in the examples a vertical dashed line is used to indicate the largest outside dimension of a member in compression). See the Design Examples in Section 8.3 and Section 8.8 for a conservative approach to handling this situation in design.

3.3.2 Indistinct distortional mode

An indistinct distortional mode, that is a situation when the distortional mode cannot immediately be identified from a minimum in the half-wavelength vs. load factor curve of a finite strip analysis, is a common problem. Part of the difficulty stems from the rather loose definition of a distortional mode, essentially stating, if it is not local and it is not global then it is distortional. A key characteristic of the distortional mode beyond the basic definition, is that in some form, distortional buckling involves the buckling of a stiffener; whether it be an edge stiffener or internal longitudinal stiffener. A small stiffener may cause interaction between local and distortional buckling and a longer stiffener between distortional and global buckling. In many cases identification of the distortional mode requires engineering judgment.

The basic options for handling an indistinct distortional mode in the finite strip context include

- refine the half-wavelengths,
- review the distortional buckling mode definitions carefully,
- create a centerline model (no rounded corners) and pin the internal fold lines to force local buckling and isolate local buckling from distortional buckling,
- use the manual elastic buckling solutions of Section 2.6 of this Guide to provide the half-wavelength in the distortional mode (L_{crd}),
- use the half-wavelength in distortional buckling for a different loading (e.g., bending instead of compression) to identify the appropriate distortional buckling values, half-wavelength only changes modestly with loading,
- vary the basic dimensions of the model slightly (typically the edge stiffener length) to recognize the trend in the distortional buckling minima, and thus identify the most appropriate half-wavelength choice, and
- if all else fails one can conservatively choose the lowest buckling mode which exhibits some of the features of the distortional buckling definition, this should be at a length greater than the local buckling half-wavelength. Theoretically, we seek a pure M_{crd} (P_{crd}); however, use of a mode with a small amount of interaction is conservative.

Also, it is important to remember that if distortional bucking (P_{crd}, M_{crd}) is above the upperbounds of Section 2.2 of this Guide then it may be safely ignored.

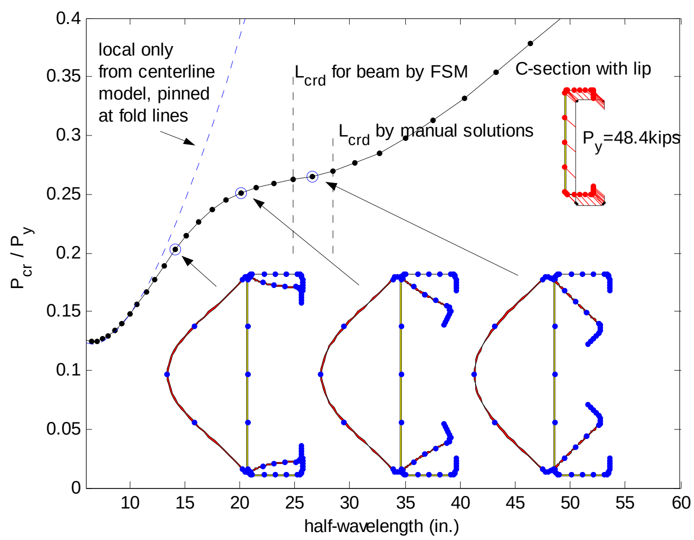


Figure 28 C-section with lip, distortional

C-section with lips: For the C-section with lips of Section 3.2.1, as shown in Figure 5(b), the distortional mode in compression is indistinct. For cross-sections with wide webs and narrow flanges this is not an uncommon occurrence. The addition of a small web stiffener, which makes the web far more efficient also removes the problem of the indistinct distortional mode, see Section 3.2.2. Figure 28 provides a closer examination of the finite strip results for the C-section and applies many of the steps given above. For half-wavelengths around 15 in., the mode shape is a mixture of local and distortional buckling, note the bending in the flange. Further, when the analysis is restricted to local buckling only (dashed curve) the P_{cr} is essentially unchanged at a 15 in. half-wavelength, indicating this is not a pure distortional mode. The Direct Strength Method design expressions were calibrated to distortional buckling modes similar to those observed between half-wavelengths of 20 to 30 in. Through variation of the lip length (not shown) and by examination of the critical half-wavelength (L_{crd}) from the manual elastic buckling solution and the beam solution (Figure 5(a)) the third of the three mode shapes identified in the inset of Figure 28 was selected as the distortional mode. Had either of the other modes shown been selected the engineer would have predicted on the conservative side. No justification exists for choosing a mode with a greater L_{crd} than that of the manual solution (Chapter 9).

Plain angle and channel: By some definition (primarily due to the half-wavelength), plain channels and plain tracks may be considered to have distortional modes. For the plain channel (track) of Section 3.2.3 and the plain angle of Section 3.2.8 see the discussion regarding indistinct local modes in the previous section (3.3.1).

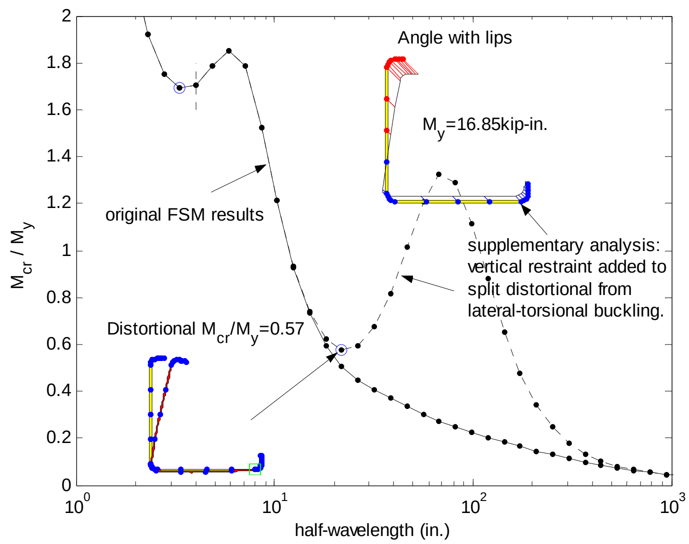


Figure 29 Angle with lips, distortional

Equal leg angle with lips: The equal leg angle with lips (Section 3.2.7) under bending, as given in Figure 17, has a distortional mode that is indistinct from the global (lateral-torsional) mode. In the strength calculations for a fully braced cross-section (Section 8.7) it is argued that this distortional mode would be restrained by the same bracing as the global mode. If this bracing is not present then the distortional mode would need to be considered in the calculation. At least two options exist (1) for the actual unbraced length visually inspect the mode shapes at half-wavelengths less than or equal to the unbraced length and assign M_{crd} to the lowest mode which has distortional characteristics, (2) add artificial restrictions in the model to remove the torsional component from distortion. This second option was performed in Figure 29, where vertical movement in the bottom right corner of the cross-section was restricted, and a distinct distortional buckling mode is readily observed.

The addition of artificial restraints to differentiate the modes does not lead to unconservative strength predictions. The purpose of the restraint is to help the engineer identify modes that are otherwise not immediately present in the analysis results. Completely ignoring a mode in a DSM calculation could lead to an unconservative strength prediction. So, determining a logical manner for including all the modes (when relevant) is important. Typically, the challenge is to identify the correct half-wavelength for selecting a given mode, and the addition of artificial restraints is often helpful in this regard. Once the proper half-wavelength is identified the elastic buckling value from the original analysis without artificial restraint at that half-wavelength may be used.

In addition to the manual elastic buckling solutions of Chapter 9 the GBT methods discussed in Section 2.5 of this Guide can provide distortional buckling solutions for C's and Z's that eliminate the problem with indistinct distortional modes. Current research is investigating mechanics-based definitions that can uniquely identify distortional modes in the finite strip method (Schafer and Adany 2005). To date, such methods, including GBT, rely on the use of centerline models of the cross-section with sharp corners only.

3.3.3 Multiple local or distortional modes (stiffeners)

As stiffeners are added to a cross-section it is possible to have more than two minima before global buckling occurs. The Direct Strength Method forces all such cases into one of two categories: local, or distortional.

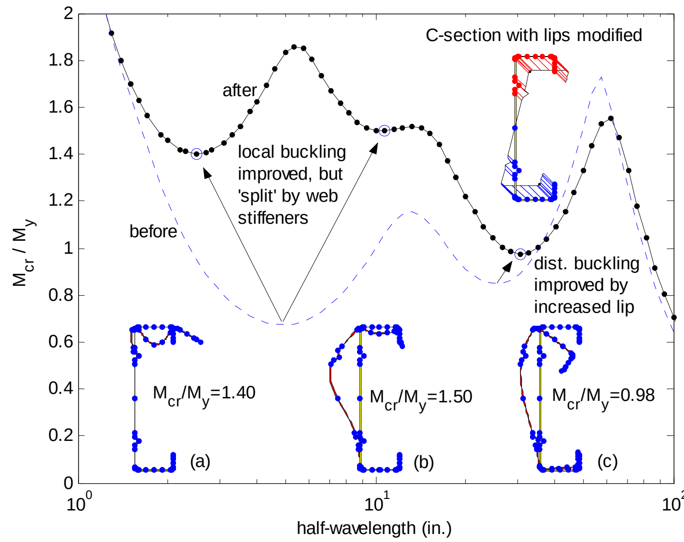


Figure 30 Modified C-section multiple modes

The C-section with lips that was modified with the addition of web stiffeners and a longer lip in Example 3.2.2 is shown again in Figure 30, this time compared with the unmodified cross-section (dashed curve). The first minimum meets the local buckling definition, as only rotation is observed at internal fold lines. The second minimum is similar to local buckling without the web stiffener, but the web stiffener itself undergoes distortional buckling. The question of whether this second minimum should be treated as local or distortional is somewhat irrelevant given that obvious local and distortional modes with lower buckling values exist. However, given the half-wavelength (relatively short) and the progression from the unmodified to the modified cross-section as shown in Figure 30, this second mode is best (though imperfectly) categorized as local buckling. The third minimum in the curve is the traditional distortional buckling mode.

For complex cross-sections the keys to conservatively and reasonably assigning elastic buckling modes to a given class (local, distortional, or global) involve slightly more than the identification of minima in the finite strip analysis results. The mode shapes of the basic cross-section, without stiffeners, should be considered. In addition, the form of the Direct Strength expressions should also be considered. For a fully braced member distortional buckling gives lower strength than local buckling, but local buckling interacts with global modes, and thus for moderate to long unbraced lengths local-global interaction typically drives the solution. If a mode occurs at relatively short half-wavelengths, or is characteristic of local buckling of the basic cross-section, then the mode should likely be categorized as local, even if it fails the strict definition of rotation only at the internal folds of the member.

3.3.4 Global modes at short unbraced lengths

Using DSM to calculate the strength of a discretely braced beam or column requires the calculation of global buckling (M_{cre}, P_{cre}). For long unbraced lengths these values may be read directly from the finite strip analysis results, similar to the methods for local or distortional buckling. For short or intermediate unbraced lengths this does not work because local or distortional buckling is the minimum mode reported in the analysis.

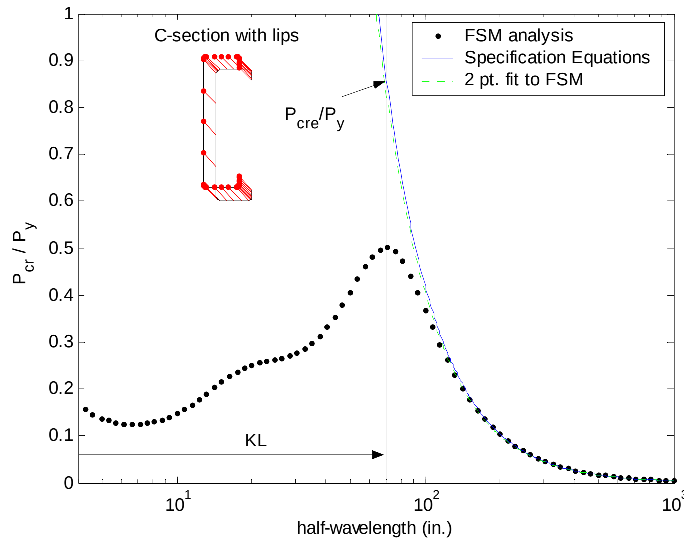


Figure 31 C-section with lips, P_{cre} at short L

Consider the C-section with lips, as shown in Figure 31 with an unbraced length 'KL' of 70 in. If one takes the raw FSM results at 70 in. ('•'s in the figure), the selected mode is a mix of distortional and flexural and P_{cre} is approximately $0.5P_y$. If instead the equations of C4.2 of the main *Specification* are employed, as detailed in Chapter 9 of this Guide, one can generate the pure global mode (flexural in this case) designated with the solid line, that has a P_{cre}=$0.85P_y$ at KL=70 in. This $0.85P_y$ as opposed to $0.5P_y$ is the correct prediction for P_{cre} to use in DSM. Since the basic form of P_{cre} is known, it is also possible to perform a simple curve fit to the FSM analysis results, shown as the dashed line predicting the same results as the *Specification* equations. This fit to FSM must employ two analysis results which display the pure mode (flexural) of interest. Use of this method is detailed in this Guide in Section 4.2 for beams and in Section 5.2 for columns, and demonstrated in the beam and column example problems of Chapter 8 (Design Examples 8.13 and 8.14).

3.3.5 Global modes with different bracing conditions

In design it is common that the basic global deformations (translation and rotation) will be braced in different ways. In the main *Specification* this situation is handled by using appropriate K factors for L_x, L_y, L_t, such that $KL_x \neq KL_y \neq KL_t$. This is important in DSM for the determination of P_{cre} or M_{cre}. In DSM the main *Specification* equations may be used to handle these bracing conditions, but FSM analysis can be used as well. Simply plot the higher modes, in addition to the first mode, as shown in Figure 32. Now use the KL appropriate to the mode under consideration and read P_{cre}/P_y directly from the plots, i.e.,

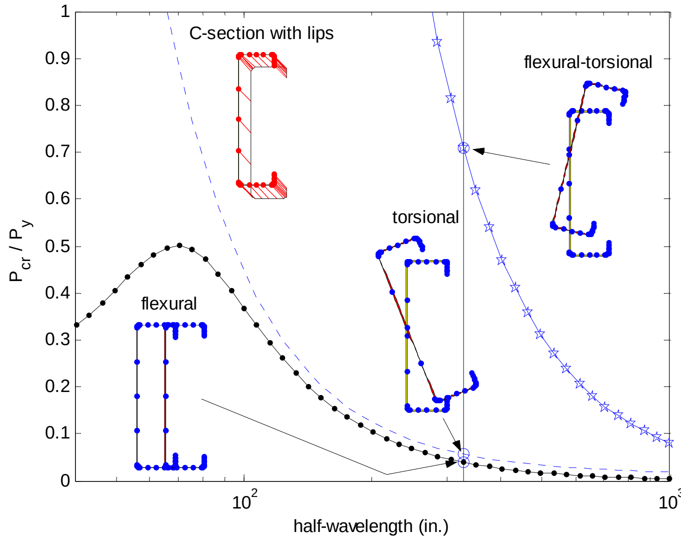


Figure 32 C-section with lips, higher modes

- KL_y for the weak-axis flexural mode,
- KL_t for the torsional mode, and
- $KL_x = KL_t$ for the torsional-flexural mode.

The finite strip analysis shows all the modes, and what bracing would be engaged as the cross-section deforms. In the example, depending on the bracing, torsion may control if only flexure is braced.

Higher modes?

In finite strip analysis, at any given half-wavelength, many modes exist. Typically the only concern is with the lowest of these modes, and these lowest modes across all half-wavelengths form the buckling value vs. half-wavelength plot () that is the heart of the finite strip analysis. However, sometimes we are interested in the higher modes that exist at a given half-wavelength. For instance, Figure 32 provides the simplest example of this idea, the first mode is global flexure the second (higher) mode is global torsion and the third (higher) mode is global torsional-flexural buckling. Higher modes also exist for local and distortional buckling at shorter half-wavelengths and are often useful when identifying indistinct modes.

3.3.6 Influence of moment gradient

For beams, local, distortional, and global buckling ($M_{cr\ell}$, M_{crd}, M_{cre}) are all potentially influenced by moment gradient. As calculated in a finite strip analysis, moment is assumed uniform. A general purpose finite element analysis (Section 2.4) could include moment gradient; for FSM the following advice is given:

Local buckling: ignore moment gradient, unless the moment changes markedly inside the short half-wavelength of a local mode, then the anticipated increase for this mode is small.

Distortional buckling: research (Yu 2005) indicates that moderate increases (30% or less) in the distortional buckling moment M_{crd} occur due to moment gradient. General purpose finite element analysis (Section 2.4) provides a rational analysis means to account for this increase. Ignoring the moment gradient is conservative.

Global (lateral-torsional) buckling: moment gradient may be accounted for using the C_b factor (Eq. C3.1.2.1-10) of the main *Specification*. Actual M_{cre}, accounting for moment gradient, is equal to $C_b M^*_{cre}$ where M^*_{cre} is the result directly from the finite strip analysis or appropriate closed-form formula as shown in Chapter 9 of this Guide.

3.3.7 Partially restrained modes

In many cases external systems (walls, sheathing, discrete braces, etc.) may partially restrain local, distortional, or global buckling. Proper inclusion of such external restraint represents a reasonable rational analysis extension (see Section 1.3.3 of this Guide) of the Direct Strength Method. If the bracing is continuous, or may be reliably approximated as continuous, then inclusion within the FSM analysis is straightforward. If bracing is discrete, or otherwise cannot be modeled in FSM, it may be possible to perform a general purpose FE analysis. Identification of the local, distortional, and global buckling modes in such a restrained case is likely to be somewhat challenging and requires engineering judgment, see Section 2.4 of this Guide for further discussion.

A typical approach to modeling additional partial restraint is the addition of an elastic spring(s) in a cross-section model, where the spring(s) represents external stiffness from the restraining system. For such a calculation to be reliable, the restraint (1) must be continuous or engaged at a fastener spacing much shorter than the half-wavelength of interest, and (2) must provide its full rotational resistance up to the nominal strength (ultimate capacity) of the member. Criterion 1 implies that local buckling rarely benefits greatly from restraint. Criterion 2 may be difficult to ensure without testing; AISI TS-1-02 as described in AISI (2002) may be a method to help provide this assurance.

Local buckling: the short half-wavelength of local buckling generally precludes consideration of external bracing. Even continuously applied sheathing/sheeting would need to have fastener spacing less than the flange width of the member to actively engage the resistance. A panel or sheathing may provide passive resistance against local buckling of a member if the buckling wave attempts to bear into the panel or sheathing. The buckling load for a plate on a tensionless, rigid, foundation shows a 30% increase in its buckling load (Shahwan and Wass 1998) due to the foundation. This provides an upper limit for the potential local buckling increase due to passive resistance of the panel or sheathing.

Distortional buckling: partial restraint can have a significant impact on distortional buckling. For example, in Figure 33 the Z-section of Section 3.2.5 is reconsidered. A rotational spring, k_ϕ, of 0.7 kip-in./rad/in. is added at mid-width of the compression flange to approximate the restraint provided by external sheathing/sheeting. Analysis of the partially restrained Z-section shows no impact on local bucking, but a significant impact on distortional buckling, and the change on global buckling is ignored here. If Example 8.5.1 is re-calculated (with k_ϕ added) M_n increases from 76 kip-in. to 87 kip-in., a 14% strength increase.

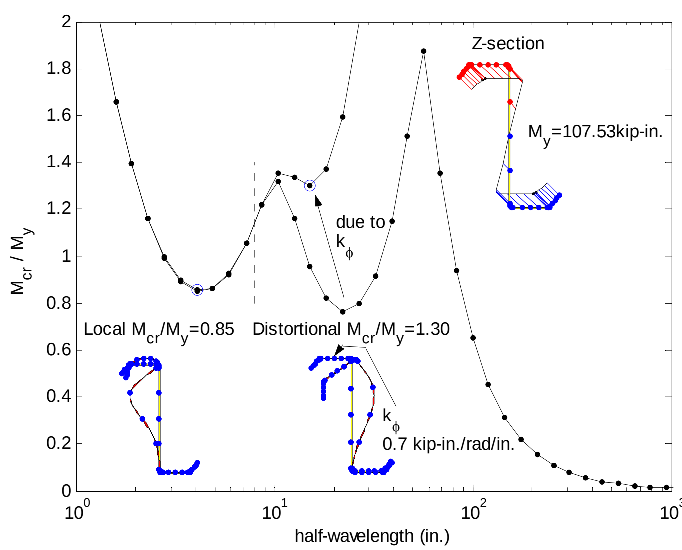


Figure 33 Z-section, mid-flange rot. spring

Global buckling: bracing can have a significant impact on global buckling modes. Continuous bracing may be directly considered in finite strip analysis, while discrete bracing is better suited for finite element models. Traditionally, restraint in global buckling modes has been handled through modifications to the boundary conditions, and engineering judgment in the selection of the effective length, KL, for the member. In cold-formed steel systems, at a minimum, KL_x, KL_y, and KL_t must be considered, this is discussed further in Section 3.3.5 of this Guide. In some instances, considering the "higher mode" response (again see Section 3.3.5) in a traditional finite strip model may provide the desired results. For example the second (higher) mode response of the hat section of Section 3.2.9 is given in Figure 34. If the torsion of this hat is restricted by bracing then buckling in the plane of the web is still possible – the second mode response provides this. Also, the results show that if first mode twist is restrained, distortional buckling should be considered. See the design examples in Section 8.9 of this Guide for further examination of this cross-section.

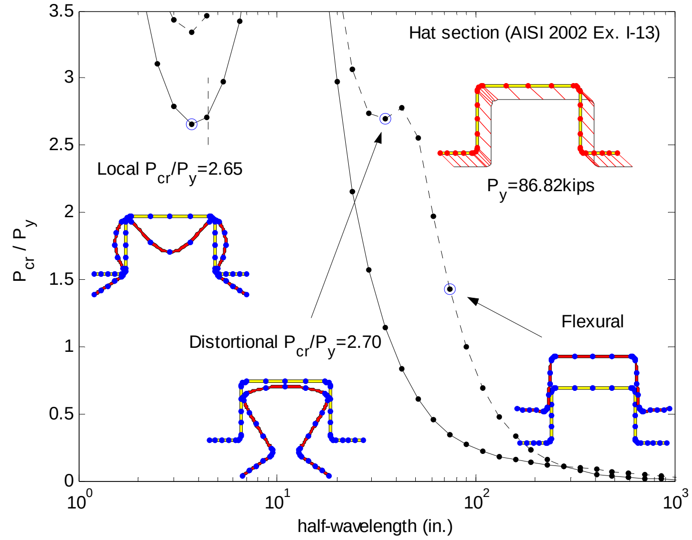


Figure 34 Hat section 2nd mode

3.3.8 Boundary conditions for repeated members

Panels, deck, and sheathing are somewhat different from other cold-formed steel members in that they are often used in a repetitive fashion. Modeling an isolated panel may be inaccurate if the connection that occurs between overlapping panels is ignored. As discussed briefly in Section 3.2.10 of this Guide with regard to a wall panel, if the edges are in compression and modeled as free (unrestrained) then buckling will likely be initiated by this free edge. One should examine such a buckling mode shape and consider if it is possible in practice. For the panel of Section 3.2.10 it was decided to also consider the case of a continuous bond between the panels to the left and right, by tying the two edges together. This results in a significant increase in the elastic distortional buckling load. The impact on the strength of the cross-section is given in the design example problems of Section 8.10.

3.3.9 Members with holes

Research extending the Direct Strength Method to members with holes is currently active. The next update to the Direct Strength Method will include explicit provisions for handling holes. The main *Specification* provisions do provide some guidance that is readily useable: in particular, when holes may be safely ignored in the strength calculation.

Ignoring holes for bending members: bending members with holes in the web are addressed in the main *Specification* in Section B2.4. Within a specific set of dimensional limits (given in B2.4), web openings may be ignored. In particular, if a mid-depth web opening is less than 38% of the web depth itself then the hole does not impact the strength. A rational extension of this finding to the Direct Strength Method is that holes do not need to be considered in calculating $M_{cr\ell}$, M_{crd}, M_{cre} if they meet the limits in main *Specification* B2.4.

Ignoring holes for compression members: compression members with holes in the web are addressed in the main *Specification* in Section B2.2. This section is applicable to widely spaced circular holes – and the hole must be less than 50% of the depth, other dimensional limits are listed in B2.2. If the element with the hole present is stocky ($\sqrt{F_y / F_{cr}} \leq 0.673$, where F_{cr} is the buckling stress of the element ignoring the hole and F_y is the yield stress) the remaining element portion is assumed to take the full yield stress. Effectively, this limits the nominal strength to $A_{net}F_y = P_{y\text{-}net}$ as opposed to $A_{gross}F_y = P_y$. In the Direct Strength Method this criteria can be rationally extended to the cross-section: if the web hole meets the geometric criteria of main *Specification* Section B2.2 and $P_{cr\ell} > 2.21P_y$ (where $P_{cr\ell}$ ignores the hole) then determine the nominal strength of the cross-section following the Direct Strength Method criteria of Section 1.2.1 (AISI 2004), but replace P_y with $P_{y\text{-}net}$ and ignore the hole in calculation of $P_{cr\ell}$, P_{crd}, P_{cre}.

Rational analysis for handling holes: Currently, for other situations, rational analysis extensions to DSM are the only alternative. For example, elastic buckling ($P_{cr\ell}$, P_{crd}, P_{cre}, $M_{cr\ell}$, M_{crd}, M_{cre}) of members with holes can be determined by general-purpose finite element analysis (Section 2.4). For example, Sarawit (2004) studied rack post cross-sections with patterned holes. He performed finite element analysis of members comprised of shell elements with the holes explicitly modeled to determine the impact of the holes on local, distortional, and global buckling. This is a time consuming process and careful visual inspection of the resulting mode shapes is required to assign local, distortional, and global buckling modes.

Take care with weighted thickness: Another engineering approach to handling holes in members is to use a weighted average thickness at the hole location. Taking care holes with this approach may not accurately capture the rigidity of the cross-section in the different buckling modes - and thus produce poor results. The weighted thickness is length dependent so different weights would be required for every half-wavelength examined in a finite strip analysis.

Take care with net section: Another engineering approach to handling holes is to model the cross-section at the net section only. Thus, the model of a C-section with a web hole would simply model the flange and the portion of the web above the hole. Such a model can lead to artificially high predictions of the local buckling stress. In the actual member, even with a hole, the center of the web may trigger the instability since longitudinal continuity exists, but in a net section model the center cannot cause the instability and the resulting buckling stress may be artificially elevated.

3.3.10 Boundary conditions at the supports not pinned

The finite strip method provides a solution for member ends which are pinned only. What should/can be done if the boundary conditions at the ends are not simply-supported?

Local buckling: typically end restraint should be ignored for local buckling. Since the half-wavelength of local buckling is short, many waves typically form inside a member, and the influence of the end conditions is quickly lost. (Hand solutions for plate buckling with different end conditions at the loaded edge do exist in the literature, but unless the length of the plate is shorter than approximately three times the plate width, the solution is the same as one with simply-supported edges.)

Distortional buckling: distortional buckling can be influenced by additional restraint provided at the ends, but no direct way exists to capture this effect in a traditional finite strip analysis. General purpose FE analysis (Section 2.4) is one recourse, GBT (Section 2.5) is another. An example of the predicted boost in P_{crd} due to fixed ends instead of pinned ends is shown in Figure 35. For global buckling at one half-wavelength, fixed ends boost the elastic buckling load four times that over pinned ends, for distortional buckling the boost is more modest: 1.6 times, for the selected cross-section. The amount of boost is cross-section dependent, and length dependent, Figure 35 provides a means to make a quick and approximate estimate to determine if more exact analysis may be warranted.

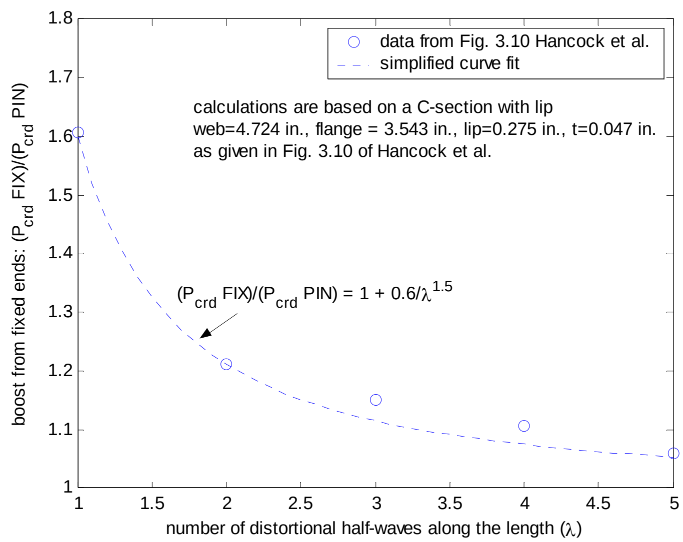
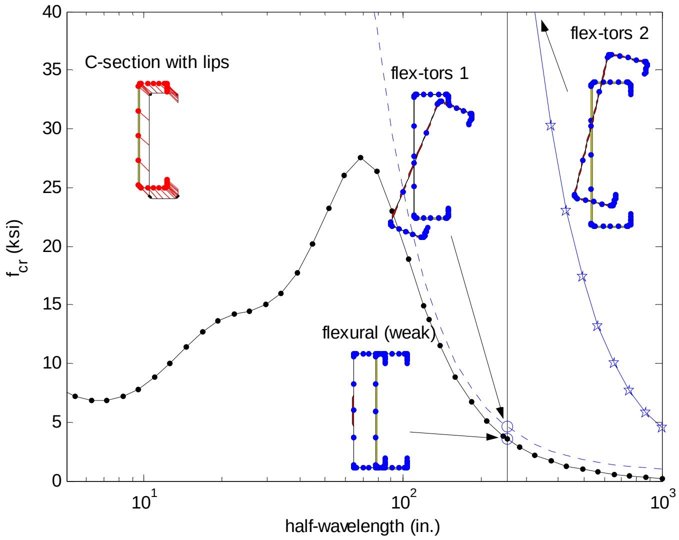


Figure 35 Distortional buckling, fixed ends

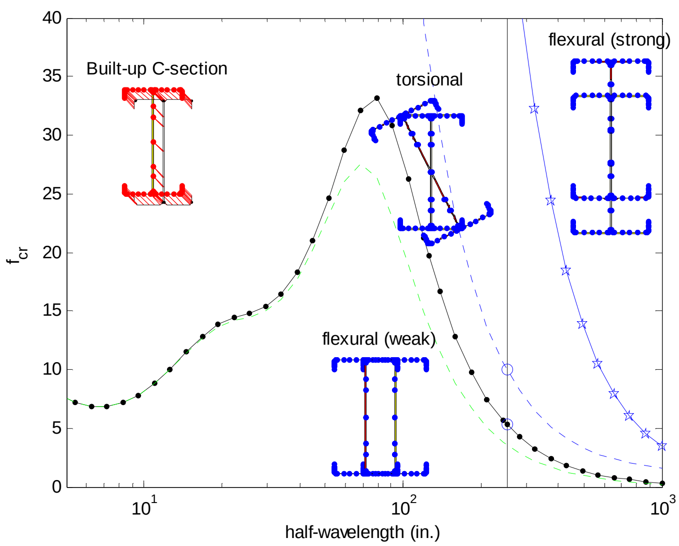
Global buckling: for boundary conditions other than pinned ends, traditional effective length factors using KL are appropriate and the elastic buckling of the global mode may be read directly from the finite strip analysis at a half-wavelength = KL. This method is detailed in Sections 3.3.4 and 3.3.5 of this Guide. Alternatively, the main *Specification* equations for column and beam buckling, as detailed in Chapter 9, can be employed. However, for point-symmetric cross-sections the *Specification* equations may be overly conservative (for M_{cre}), and for unsymmetric cross-sections the main *Specification* provides no provisions, although the AISI (2002) *Design Manual* Part V Section 3 provides a series of lengthy derivations that can be used.

3.3.11 Built-up cross-sections

Built-up cross-sections may be approximately modeled in finite strip analysis. The analysis is similar to the discussion in Section 3.3.7 of this Guide on partially restrained modes. Fasteners may be modeled as elements connecting two members together, or two parts of a model may be constrained to act identically (i.e., the "ideal" continuous fastener).



(a) Finite strip results in pure compression for C-section with lips (Section 3.2.1)



(b) Finite strip results for built-up C-sections,
webs are discretely connected 3 in. above and below centerline

Figure 36 Comparison of single and built-up cross-section

For example, Figure 36 provides the results for the 9CS2.25x059 modeled on its own, and a back-to-back built-up cross-section modeled with ideal fasteners connecting the web together at two points, 3 in. above and below centerline. This is an upperbound model, because the connection is assumed (a) continuous and (b) axial and shear are both connected. This built-up cross-section is motivated from AISI (2002) *Design Manual* Example II-8 which is discussed further in Chapter 5. The results have been plotted in terms of stress so that the two cross-sections may be most readily compared to one another. The buckling stress for local and distortional buckling are unaffected by connecting the two cross-sections together. Thus, $P_{cr\ell}$, P_{crd} are simply twice the single cross-section value in the built-up cross-section. However, in the built-up cross-section the weak-axis flexural mode is increased, and the torsional-flexural mode is replaced by a separate torsion mode and a strong-axis flexure mode. If the weak-axis flexure is adequately braced, then these modes become the potential P_{cre} modes of interest. (Note, if the fasteners cannot provide the shear resistance then the torsional-flexural mode will still occur).

Note, a toe-to-toe built-up C-section will have a different behavior from the back-to-back cross-section. Most noticeably, the distortional buckling (P_{crd} or M_{crd}) will be elevated greater than just two times a single C-section response. Torsional-flexural buckling will also be greatly increased due to the enhanced torsional stiffness of the closed cross-section shape.

4 Beam design

This chapter begins with the application of the Direct Strength Method to fully braced beams. To illustrate the influence of unbraced length on bending strength, solutions are provided for generating a beam chart in Section 4.2. This is followed by Section 4.3 which focuses on serviceability calculations using the Direct Strength Method. Section 4.4 addresses shear, web crippling, and interaction checks – which are largely outside the scope of the Direct Strength Method and require using the main *Specification* in conjunction with the Direct Strength Method. Finally, in Section 4.5, the beam design examples from the AISI (2002) *Design Manual* are considered and updated for use with the Direct Strength Method. Issues of interest that arise in these examples include the use of C_b, R for uplift, and cold work of forming. Chapter 8 provides Direct Strength Method beam design examples for the 12 different cross-sections of Section 3.2.

4.1 Beam design for fully braced beams

In the main *Specification* it is common to consider the nominal strength of a fully braced beam and determine the effective section modulus for this case (i.e., $M_n=S_eF_y$). This provides the most conservative approximation of the effective properties, and Chapter 1 of the AISI (2002) *Design Manual* provides extensive coverage of this calculation.

The nominal strength of a fully braced member in the Direct Strength Method can be readily found by setting the global buckling capacity to its full nominal strength, the yield moment (i.e., $M_{ne}=M_y$ of DSM Section 1.2.2.1), and then proceed normally through the Direct Strength Method expressions. Thus, the definition of a fully braced beam assumes that the bracing precludes global modes, but leaves local and distortional buckling free to form. Examination of the buckling mode shapes themselves can help in determining the impact that bracing may have.

The bending strength for fully braced beams is provided for all the cross-sections of Section 3.2 and in Chapter 8, Examples 8.1 through 8.12. For general beam design (not fully braced) the primary difference is that lateral-torsional buckling must now be considered ($M_{cre} < 2.78M_y$ implying $M_{ne} < M_y$) and therefore the local buckling strength ($M_{n\ell}$) will be reduced. In addition, shear, web crippling, and interaction of bending + shear, and bending + crippling, must also be considered. These issues are discussed further in the following sections.

Fully braced?

Fully braced. A cross-section that is braced such that global buckling is restrained. The term fully braced is used extensively in the discussion and Design Examples of this Guide. This is largely due to a desire to compare Direct Strength Method results to the effective property calculations used in the main *Specification* (e.g., see Part I of AISI (2002)). A key difference between the main *Specification* and the Direct Strength Method that influences this comparison is that DSM includes the possibility of distortional buckling when a member is fully braced.

4.2 Beam charts, local, distortional, and global buckling as a function of length

One of the main advantages of DSM is the ability to examine cross-section variations with efficiency. Whether you are a product designer, or a consulting engineer, beam charts which provide the nominal strength of a given beam vs. length are often useful in design. To produce a beam chart with DSM correctly it is important to understand the unique influence of unbraced length on each of the buckling classes: local, distortional, and global. For creating the beam chart, M_y, $M_{cr\ell}(L)$, $M_{crd}(L)$, $M_{cre}(L)$, where (L) indicates the quantity as a function of length, are required.

Local buckling occurs at short half-wavelengths, thus in longer members it merely repeats itself many times along the length. Once the length of the member is greater than the depth (or largest dimension) of the cross-section little if any change occurs in the local buckling value. Given this behavior it is prudent to assume that

$$M_{cr\ell}(L) = M_{cr\ell}\ ,$$

i.e., local buckling remains unchanged for any length.

Distortional buckling occurs at an intermediate half-wavelength and an unbraced length shorter than the distortional buckling half-wavelength length is possible. In limited cases one can read a distortional buckling value at L less than the half-wave length corresponding to the distortional buckling minimum point directly from the finite strip analysis results. Alternatively, a shortened length may be substituted directly in the closed-formed distortional buckling formulas provided in Chapter 9 of this Guide. The simplest procedure is to use an approximation based on the work of Yu (2005):

$$M_{crd}(L < L_{crd}) = M^{*}_{crd}(L/L_{crd})^{\ln(L/L_{crd})}, \text{ and}$$

$$M_{crd}(L \geq L_{crd}) = M^{*}_{crd}\ ,$$

where L is the length of interest, L_{crd} is the half-wavelength at which the distortional buckling moment is a minimum (e.g., in a finite strip analysis) and M^{*}_{crd} is the distortional buckling moment at the minimum point. The equations above are accurate for a wide range of C- and Z-sections (Yu 2005) and are recommended for application for any cold-formed steel cross-section.

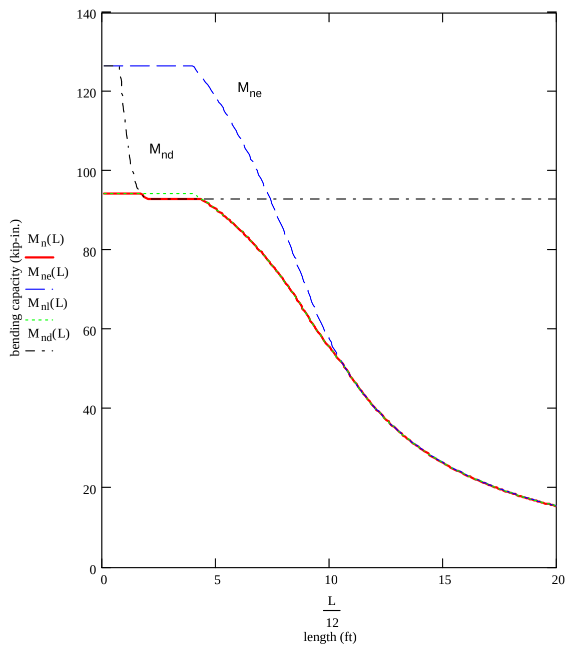
Global (lateral-torsional) buckling occurs at long lengths. To determine $M_{cre}(L)$ two options are available: (1) use the closed-form expression in the main *Specification* as illustrated in Section 2.6 of this Guide, or (2) recognize the form of M_{cre} as a function of L and fit an appropriate expression to the generated finite strip data (see Section 3.3.4 of this Guide for further discussion). Method (2) does not require the calculation of cross-section properties and is relatively easily implemented. The form of M_{cre} as a function of L is known:

$$M_{cre}{}^2 = \alpha(1/L)^2 + \beta(1/L)^4$$

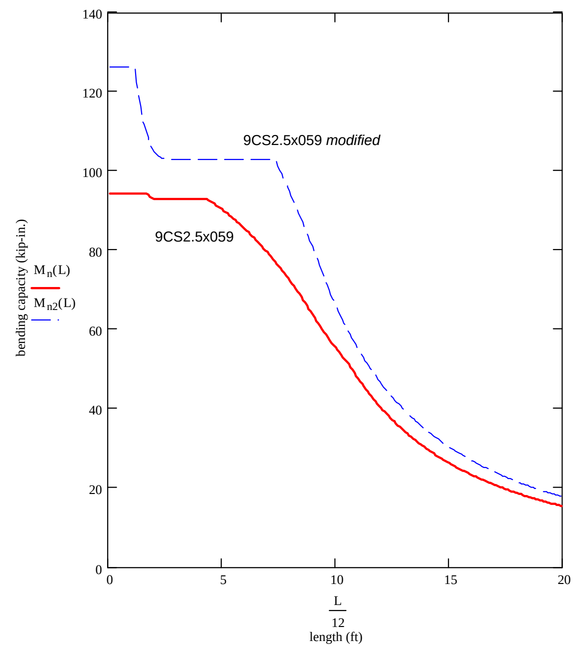
To find α and β pick any two pairs of points in the finite strip analysis curve at half-wavelengths long enough that the mode shapes display the lateral-torsional mode. Defining such pairs as (L_{cr1},M_{cre1}) and (L_{cr2},M_{cre2}), then α and β are:

$$\alpha = \frac{M^2_{cre1}L^4_{crl} - M^2_{cre2}L^4_{cr2}}{L^2_{cr1} - L^2_{cr2}} \qquad \beta = \frac{(M^2_{cre1}L^2_{cr1} - M^2_{cr2}L^2_{cr2})L^2_{cr1}L^2_{cr2}}{L^2_{cr2} - L^2_{cr1}}$$

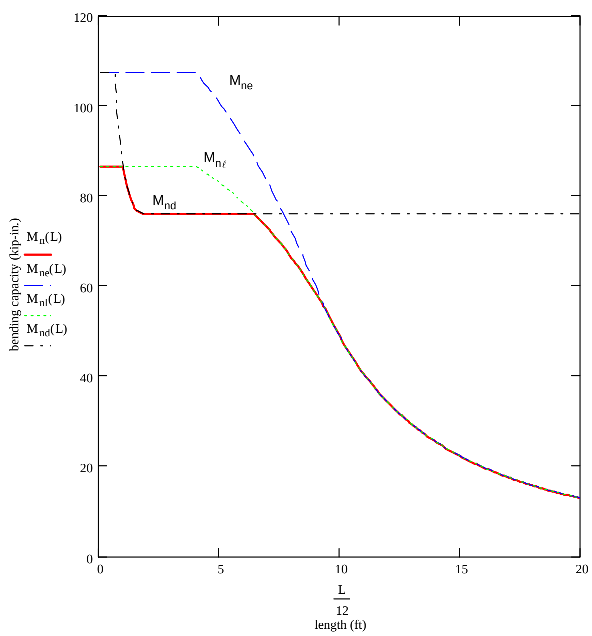
Following this methodology a beam chart is developed for the C-section with lips, of Section 3.2.1. The complete development of the chart is given in Section 8.13 of this Guide. The final chart is provided in Figure 37(a) and (b). Additional charts for the Z-section with lips are provided in parts (c) and (d) of this figure.



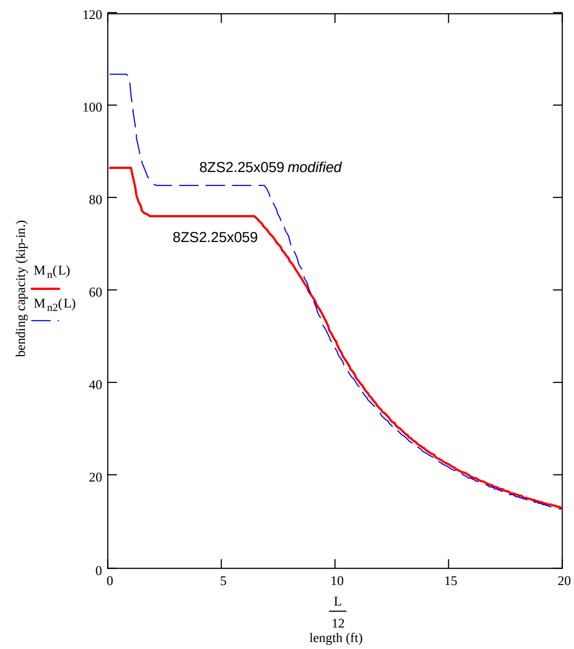
(a) M_n for C-section with lips
(see Sections 3.2.1, 8.1 and 8.13)



(b) M_n for C-section with lips and modified
(see Sections 3.2.2, 8.2 and 8.13)



(c) M_n for Z-section with lips
(see Sections 3.2.5 and 8.5)



(d) M_n for Z-section with lips and modified
(see Sections 3.2.6 and 8.6)

Figure 37 Beam charts for C- and Z-sections with lips by the Direct Strength Method

Figure 37(a) and (c) show how the three Direct Strength prediction equations behave as a function of length: $M_{n\ell}$ is reduced from M_{ne}, unless M_{ne} is low enough at which length $M_{n\ell}$ and M_{ne} converge. M_{nd} is a separate strength check that is independent of length for intermediate to large lengths, and may control the strength over a relatively short regime of unbraced length. The modified cross-sections provide improved performance over the original cross-sections.

4.3 Deflections and serviceability

For determination of deflections in a serviceability check, in the main *Specification*, effective properties of a member are determined at the service stress level of interest. The procedure for calculating the effective properties is identical to a strength calculation except the maximum stress is the service stress.

The Direct Strength Method essentially uses a similar philosophy as the main *Specification*, but since the equations are in terms of strength, the implementation is more awkward. The service level moment (M) is used as the peak moment (i.e., M replaces the yield moment M_y) and the deflection strength M_d of the cross-section is determined. The ratio of these two moments provides an approximate reduction in the stiffness of the member at the service moment, M.

The specific expressions for determining the reduced moment of inertia are given in DSM Eq. 1.1.3-1 (Appendix 1, AISI 2004). An example deflection calculation for the C-section with lips of Section 3.2.1 is provided in Example 8.1.3 of this Guide. This example also provides a chart of the reduced stiffness as a function of the service level moment, M. This chart is reproduced in Figure 38 below.

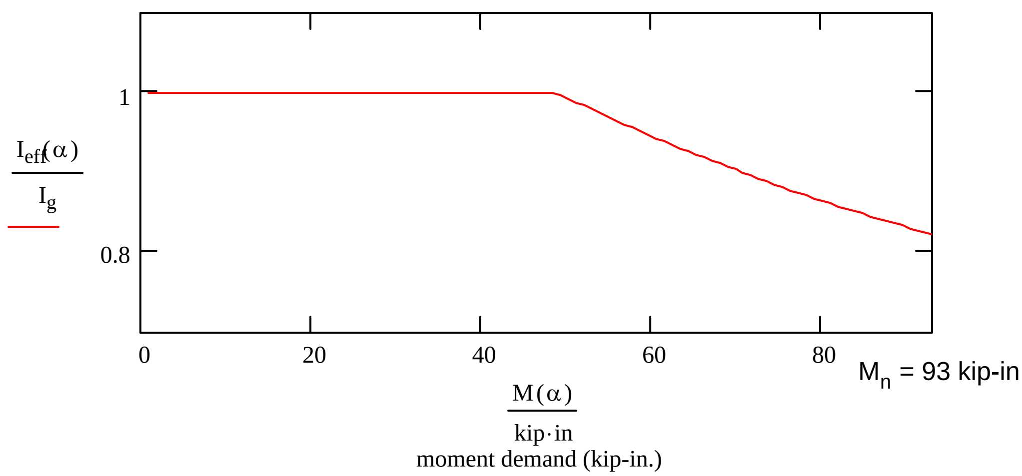


Figure 38 Reduced stiffness as a function of service moment for 9CS2.5x059 of Sections 3.2.1 and 8.1

4.4 Combining DSM and the main *Specification* for beams

While DSM does provide a general methodology for prediction of the nominal flexural strength, M_n, strength in shear, and web crippling are not covered. As a result, the main *Specification* is required for shear, and web crippling calculations when applicable.

4.4.1 Shear

The provisions for shear are in the main *Specification* Section C3.2. For members with flat webs these provisions can be used without modification.

For members which do not have flat webs no guidance is provided in the *Specification*. As a rational analysis extension the existing Section C3.2.1 equations are recast into the Direct Strength format and are suggested for use:

for $\lambda_v \leq 0.815$

$$V_n = V_y$$

for $0.815 < \lambda_v \leq 1.231$

$$V_n = 0.815\sqrt{V_{cr} V_y}$$

for $\lambda_v > 1.231$

$$V_n = V_{cr}$$

where $\lambda_v = \sqrt{V_y / V_{cr}}$

$$V_y = A_w 0.60 F_y$$

V_{cr} = critical elastic shear buckling force

For members with flat webs these expressions yield the same results as the existing main *Specification* Section C3.2. As a rational analysis extension, ϕ=0.8, and Ω=2.0 per A1.1(b) of the main *Specification*. V_{cr} could be determined by FE analysis (Section 2.4) or other methods.

Holes: For shear in members with holes the main *Specification* C3.2.2 provides a method for members meeting given geometric limits. For further information on members with holes see Section 3.3.9 of this Guide.

4.4.2 Combined bending and shear

The main *Specification* provisions of Section C3.3 provide a means to consider combined bending and shear. As detailed in Table 1, M_n calculated by the Direct Strength Method may be used in C3.3.

4.4.3 Web crippling

Web crippling is covered in Section C3.4 of the main *Specification*. The provisions are empirically derived from experimental data, so rational analysis extensions without testing are difficult. Basic geometric limits for the applicability of the provisions to I, C, Z, hat, and deck cross-sections are provided. For other cross-sections strength must be determined by rational analysis or testing (section A1.1 of the main *Specification*).

4.5 Notes on example problems from AISI (2002) *Design Manual*

The AISI (2002) *Design Manual* includes six beam design examples. Those examples are used as the basis for the beam examples provided in this Guide (Chapter 8). To aid the reader in completing a comparison of the traditional main *Specification* methods to DSM, Table 2 was prepared. Within the design examples of this Guide additional commentary comparing the main *Specification* approach with DSM is provided.

Table 2 Comparison of beam examples between the AISI (2002) *Design Manual* and this Guide

AISI (2002) *Design Manual* Example	DSM Design Guide Example
II-1 Four Span Continuous C-Purlins Attached to Through Fastened Roof – LRFD	Section 8.1 Design Examples 8.1.1 - 8.1.3
II-2 Four Span Continuous Z-Purlins Attached to Through Fastened Roof – ASD	Section 8.5 Design Examples 8.5.1 and 8.5.2
II-3 C-Section Without Lips Braced at Mid-Span	Section 8.3 Design Examples 8.3.1 and 8.3.2
II-4 Fully Braced Hat Section	Section 8.9 Design Example 8.9.1
II-5 Tubular Section – Round	Not covered by the Direct Strength Method[1]
II-6 C-Section with Openings	Not covered in this Guide[2]

1 This cross-section is not covered by the Direct Strength Method, the main *Specification* rules apply.

2 No design example is provided in this Guide; however two items worthy of discussion are provided here.

(a) ***Cold work of forming***: AISI (2002) Example II-6 uses the cold work of forming provisions of A7.2 of the main *Specification*. Main *Specification* Section A7.2 states that for all effective width calculations ρ must equal one when calculated at the average, elevated, yield stress $F_y = F_{ya}$. In the Direct Strength Method ρ is not used, instead the $\rho = 1$ check is equivalent to ensuring $M_{n\ell} = M_y$ at $F_y = F_{ya}$, and $M_{nd} = M_y$ at F_y=F_{ya}. Cross-sections which meet this criterion are eligible for inclusion of cold work of forming and use of F_y=F_{ya} in DSM.

(b) ***Bending (with holes)***: For the example cross-section $d_o/h = 1.5/3.643 = 0.412 > 0.38$, therefore the hole may not be ignored in the calculation. Since the hole may not be ignored, simple rational analysis extension of the Direct Strength Method becomes complicated. The AISI (2002) *Design Manual* Example proceeds by calculating the effective section for the remaining lip above the hole – this stress (F_{cr} in the AISI (2002) Example II-6) could be used to approximate $M_{cr\ell}$ in the Direct Strength Method, but a similar calculation for M_{crd} is not immediately available, see Section 3.3.9 of this Guide for further discussion.

5 Column design

This chapter begins with a discussion of the strength determination of braced columns. Although it is somewhat artificial to divorce columns from beam-columns this Chapter covers only concentrically loaded columns, and leaves beam-columns to Chapter 6. To illustrate the influence of unbraced length on compressive strength, solutions are provided for generating a column chart in Section 5.2. In Section 5.3, the design examples from Part III of the AISI (2002) *Design Manual* are considered and updated for use with the Direct Strength Method. Part III of the *Manual* addresses both columns and beam-columns; in this Chapter only the column calculations will be considered. Issues of interest that arise in these examples include bracing boundary conditions and constrained buckling.

5.1 Column design for continuously braced columns

In the main *Specification* it is common to consider the nominal strength of a braced column and determine the effective area for this case (i.e., $P_n=A_eF_y$). This provides the most conservative approximation of the effective area, and Part I of the AISI (2002) *Design Manual* provides extensive coverage of this calculation.

The nominal strength of a fully braced member in the Direct Strength Method can be readily found by setting the global buckling capacity to its full nominal strength, the squash load (i.e., $P_{ne}=P_y$ of DSM Section 1.2.1.1), and then proceeding normally through the Direct Strength Method expressions. Thus, the definition of a fully braced column assumes that the bracing precludes global buckling modes, but leaves local and distortional buckling free to form. Examination of the buckling mode shapes themselves can help in determining the impact that bracing may have.

The compressive strength for braced columns is provided for all the cross-sections of Section 3.2 in Chapter 8, Examples 8.1 through 8.12.

For general column design (not fully braced) the primary difference is that flexural, or torsional-flexural buckling must now be considered and therefore the local buckling strength ($P_{n\ell}$) will be reduced. These issues are discussed further in the following sections.

5.2 Creating column charts

Creation of a column chart is similar to that of a beam chart (Section 4.2). The key consideration is to understand the unique influence that the unbraced length has on each of the buckling classes: local, distortional, and global. For creating a column chart, P_y, $P_{cr\ell}(L)$, $P_{crd}(L)$, and $P_{cre}(L)$, where (L) indicates quantity as a function of length, are required.

Local buckling occurs at short half-wavelengths, thus in longer members it merely repeats itself many times along the length. Once the length of the member is greater than the depth (or largest dimension) of the cross-section, little if any change occurs in the local buckling value. Given this behavior it is prudent to assume that

$$P_{cr\ell}(L) = P_{cr\ell}$$

i.e., local buckling remains unchanged for any length.

Distortional buckling occurs at an intermediate half-wavelength and an unbraced length shorter than the distortional buckling half-wavelength length is possible. In limited cases one can determine a distortional buckling value at L less than the half-wave length corresponding to the distortional buckling minimum point directly from the finite strip analysis results. Alternatively, a shortened length may be substituted directly in the closed-formed distortional buckling formulas detailed in Chapter 9 of this Guide. The simplest procedure is to use an approximation based on the work of Yu (2005):

$$P_{crd}(L < L_{crd}) = P^*_{crd}(L/L_{crd})^{\ln(L/L_{crd})} \text{ and}$$

$$P_{crd}(L \geq L_{crd}) = P^*_{crd}$$

where L is the length of interest, L_{crd} is the half-wavelength at which the distortional buckling load is a minimum (e.g., in a finite strip analysis) and P^*_{crd} is the distortional buckling load at the minimum point. This approximate expressions was derived for C- and Z-sections (Yu 2005) but it accurate enough for general applicability.

Global (flexural, torsional-flexural) buckling occurs at long lengths. To determine $P_{cre}(L)$ two options are available: (1) use the closed-form expression in main *Specification* Sections C4.1-C4.4 as illustrated in Chapter 9, or (2) recognize the form of P_{cre} as a function of L and fit an appropriate expression to the generated finite strip data (see Section 3.3.4 for further discussion). Method (2) has the advantage in that it does not require the calculation of cross-section properties and is relatively easily implemented. The form of P_{cre} as a function of L is known:

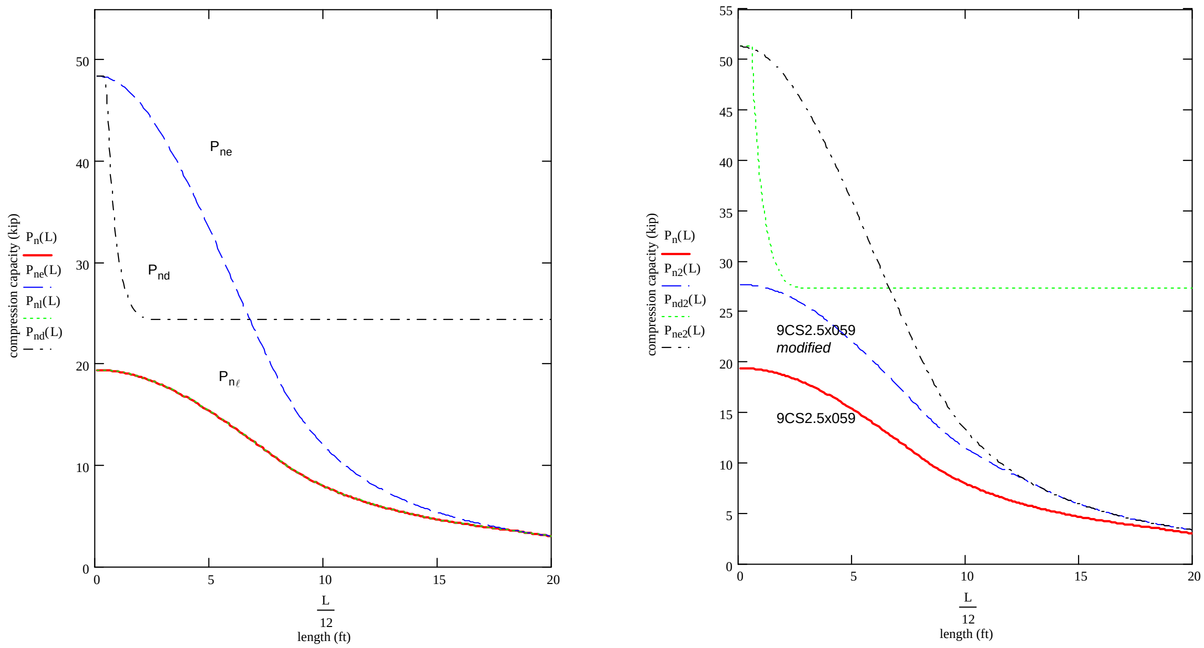
$$P^*_{cre} = \alpha(1/L)^2 + \beta(1/L)^4$$

To find α and β which can be determined by picking any two pairs of points in the finite strip analysis curve at lengths long enough that clearly display the flexural or torsional-flexural mode of interest, namely if we define the pairs (L_{cr1},P_{cre1}) and (L_{cr2},P_{cre2}), then α and β are:

$$\alpha = \frac{P^2_{cre1}L^4_{cr1} - P^2_{cre2}L^4_{cr2}}{L^2_{cr1} - L^2_{cr2}} \qquad \beta = \frac{(P^2_{cre1}L^2_{cr1} - P^2_{cre2}L^2_{cr2})L^2_{cr1}L^2_{cr2}}{L^2_{cr2} - L^2_{cr1}}$$

Columns may display different buckling modes at long lengths (e.g., switching from torsional-flexural to flexure) it is important that the two selected points be consistent with the buckling mode of interest. Further, it may be desirable to determine curves for more than one mode – i.e., for both weak-axis flexure, and for torsional-flexural buckling.

Following this methodology a column chart was developed for the C-section with lips of Section 3.2.1. The complete development of the chart is given in Section 8.14 of this Guide. The final chart is provided in Figure 39.



(a) P_n for C-section with lips (see Sections 3.2.1, 8.1, and 8.14)

(b) P_n for C-section with lips and modified (see Sections 3.2.2, 8.2, and 8.14)

Figure 39 Column chart for C-section with lips by Direct Strength Method

Figure 39 shows how the three Direct Strength prediction equations behave as a function of length: $P_{n\ell}$ reduces P_{ne}, except at long lengths where P_{ne} is reduced far enough that $P_{n\ell}$ and P_{ne} converge. P_{nd} is a separate strength check that is independent of length for intermediate to large lengths, but in these cross-sections never controls. Figure 39(b) shows that the modified cross-section provides improved performance over the original cross-section.

5.3 Notes on example problems from the 2002 AISI *Manual*

The AISI (2002) *Design Manual* includes six column design examples. Those examples are used as the basis for the column examples provided in this Guide (Chapter 8). To aid the reader in completing a comparison of the traditional main *Specification* methods to DSM, Table 3 was prepared. Within the design examples of this Guide additional commentary comparing the main *Specification* approach with DSM is provided. Several of the design examples in Part III of the AISI (2002) *Design Manual* cover beam-column design. Only the compressive behavior is examined here. Beam-columns are treated in Chapter 6.

Table 3 Comparison of column examples between the AISI (2002) *Design Manual* and this Guide

AISI (2002) *Design Manual* Example	DSM Design Guide Example
III-1 Braced C-Section With Lips	Section 8.1 Design Examples 8.1.4 and 8.1.5
III-2 C-Section With Lips With Holes	Not covered in this Guide[1]
III-3 Sheathed Stiffened C-Stud	AISI-COFS provisions now apply[2]
III-4 Unbraced Equal Leg Angle With Lips	Section 8.7 Design Examples 8.7.3 and 8.7.4
III-5 Tubular Section Round	Not covered by the Direct Strength Method[3]
III-6 Stiffened Z-Section With One Flange Through-Fastened to Deck or Sheathing	Section 8.5 Design Examples 8.5.3 and 8.5.4
III-7 Hat Section	Section 8.9 Design Examples 8.9.2 and 8.9.3
III-8 I-Section – Built Up from Channels	No design example provided[4]

1 No design example is provided in this Guide; however two items worthy of discussion are provided here.

- ***(a)*** ***Torsional-flexural buckling***: In the AISI (2002) Example III-2, K_xL_x=108 in., but K_tL_t=54 in., because torsional bracing is provided at mid-height. This situation is handled conveniently in the main *Specification* equations of C4.2, but not in the finite strip method (see Section 3.3.5 of this Guide). Use of the C4.2 equations in this manner ($K_xL_x \neq K_tL_t$) is an approximation. A rational analysis using a general purpose finite element analysis (see Section 2.4 of this Guide) will yield a more accurate solution.
- ***(b)*** ***Holes***: the method employed in AISI (2002) Example III-2 combines Sections D4 and B2.2 of the main *Specification*. The procedure employed, determining the effective width of the plate above and below the holes, is not readily extendable to the Direct Strength Method.

2 This design example relies extensively on Section D4 of the AISI (2001) *Specification*. This section was extensively modified (and reduced) in the 2004 Supplement to the AISI *Specification*. Most of the provisions in Section D4 have been removed. A new standard, *Standard for Cold-Formed Steel Framing - Wall Stud Design* (AISI 2004b) has been developed to replace this material.

3 This cross-section is not covered by the Direct Strength Method, the main *Specification* rules apply.

4 The cross-section used in this example is covered in Section 3.2.1 of this Guide. Elastic buckling of built-up cross-sections is discussed in Sections 3.3.7 and 3.3.11 of this Guide. The main *Specification* provides specific provisions (C4.5) for determining the slenderness of built-up cross-sections in global buckling modes that place the fasteners in shear as used in AISI (2002) Example III-8. Alternatively, a model may be created directly in a finite strip analysis (see Section 3.3.11 of this Guide). For the compressive strength see Design Example 8.1-5 for determining the strength of a single C-section once the correct global P_{cre} is known. In this back-to-back built-up cross-section $P_{cr\ell}$, P_{crd} and P_y are simply twice their value as a single C-section.

6 Beam-column design

Conventional beam-column design, following the basic methodology of the main *Specification*, is a simple extension of the Direct Strength Method. This chapter provides a summary of this basic method (Section 6.1) and covers several example problems applying the method (Section 6.2). As in any beam-column design once the basic beam and column strength is established the chief complication is accurately determining the amplified, or second-order, bending moment demand (this bending moment demand is the required flexural strength). Finally, in Section 6.3 of this Guide a future method for beam-column design using the Direct Strength Method is previewed. While this method is still under the development, it provides another means to understand the advantages of moving towards direct analysis of stability as in the Direct Strength Method. A design example is also provided.

6.1 Main *Specification* methodology

The basic interaction equation, for example in ASD format, Eq. C5.2.1-1, is as follows:

$$\frac{\Omega_c P}{P_n} + \frac{\Omega_b C_{mx} M_x}{M_{nx}\alpha_x} + \frac{\Omega_b C_{my} M_y}{M_{ny}\alpha_y} \leq 1.0$$

where:

- P, M_x and M_y: The first-order required strengths (demands). P, M_x and M_y, axial load, and bending moment about the x and y axes respectively are determined from conventional linear elastic analysis.
- P_n: the nominal compressive strength. P_n with related safety factor Ω_c is detailed for the Direct Strength Method in Chapter 5.
- M_n: the nominal flexural strength about x or y with related safety factor Ω_b as detailed for the Direct Strength Method in Chapter 4.
- C_m: the moment gradient factor. C_m (about x or y) accounts for the case where the primary (first-order) moment and the second-order amplifications do not occur at the same location in the cross-section. The method for determination is fully addressed in the main *Specification* and does not change with the Direct Strength Method.
- α: the moment amplification factor. α (about x or y) is $1-\Omega_c P/P_E$ as described in the main *Specification*. P_E is the elastic buckling load of the cross-section about the same axis as the primary bending moment, i.e., for strong axis moment M_x, global buckling load P_E is P_{Ex}. Global buckling loads may be determined from the main *Specification* equations or directly from a finite strip analysis.

The auxiliary interaction equation, for example in ASD format, Eq. C5.2.1-2:

$$\frac{\Omega_c P}{P_{no}} + \frac{\Omega_b M_x}{M_{nx}} + \frac{\Omega_b M_y}{M_{ny}} \leq 1.0$$

uses all of the same terms, except,

- P_{no}: the nominal axial strength ignoring global buckling. P_{no} may also be determined in the Direct Strength Method by setting $P_{ne}=P_y$ as discussed in Chapter 5 and demonstrated in the design examples of Chapter 8.

LRFD uses a different format, but the calculation procedure is essentially the same.

6.2 Design examples

Four design examples for beam-column strength (capacity) are provided in Chapter 8. The first three are based on the AISI (2002) *Design Manual:* Example III-1, Example III-4, and Example III-7. To aid the reader in completing a comparison of the traditional main *Specification* methods to DSM, Table 4 was prepared. Within the Design Examples of this Guide additional commentary comparing the main *Specification* approach with DSM is provided.

Table 4 Comparison of beam-column examples between AISI (2002) *Design Manual* and this Guide

AISI (2002) *Design Manual* Example	DSM Design Guide Example
III-1 Braced C-Section With Lips	Section 8.1 Design Example 8.1.6
III-4 Unbraced Equal Leg Angle With Lips	Section 8.7 Design Example 8.7.5
III-7 Hat Section (ASD)	Section 8.9 Design Example 8.9.4

The fourth example covers beam-column design of a simply-supported 550T125-54 track section. The member is 49.3 in. long. The loading is a concentric axial load and a uniform moment placing weak-axis bending demand on the track section (flange tips in compression). The cross-section used in this example is covered in Section 3.2.3 of this Guide. The nominal strength of this cross-section is examined in Section 8.3 Design Example 8.3.6 in this Guide.

6.3 Future directions for DSM: Direct analysis of beam-columns

The advantage of the Direct Strength Method is that the stability of the entire cross-section under a given axial load (P) or bending moment (M) is completely investigated. Local, distortional, and global buckling of the column or beam is explored. It is natural to extend this idea to the stability of the cross-section under any given P and M combination. Where, now, the three buckling modes: local, distortional, and global buckling are explored under the actual P and M combination of interest, instead of separately for P and separately for M. Such an analysis can lead to far different behavior than typically assumed in the interaction equation approach used in the main *Specification*.

The fundamental difference between the interaction equations and a more thorough stability analysis can be understood by answering a simple question: *for all cross-sections does the maximum axial capacity exist when the load is concentric*? The interaction equation approach says, yes, any additional moment caused by a load away from the centroid will reduce the nominal strength of the cross-section. While a conservative answer, it is not always correct. If moving the axial load causes the relative compressive demand on a weak part of the cross-section to be relieved the cross-section strength will benefit from this. Interaction diagrams make some sense for determining when a simple cross-section yields, but stability, this is another matter.

Research is currently underway to take advantage of performing direct analysis of a cross-section. To avoid overwhelming the reader of this Guide with laborious details, a single example was created to demonstrate the potential of a direct analysis of a beam-column. The example is for a track section and is provided in Section 6.3.1 of this Guide. Comparison with Example 8.3.6 in Section 8.3allows for a direct contrast with conventional methods.

The goal of current research is to provide an easy to create, unique interaction equation for all cross-sections. This method will reflect changes in first yield for unsymmetric cross-sections, and the direct determination of the stability of cross-sections under multiple loads. For now, the example in the following section provides a preview of the potential power of the method and demonstrates ongoing research directions.

6.3.1 Direct analysis beam-column design strength example

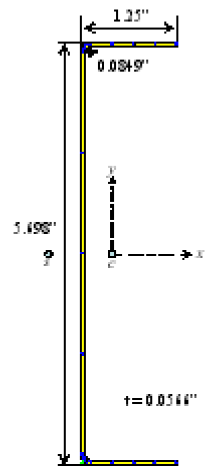
The following pages provide a design example using the method proposed in Section 6.3 above. The cross-section selected is the plain channel of Section 3.2.3. Section 8.3 Design Examples 8.3.1 through 8.3.6 provide conventional strength calculations for this member and are referenced as needed in the example.

6.3.1 Direct analysis beam-column design strength example

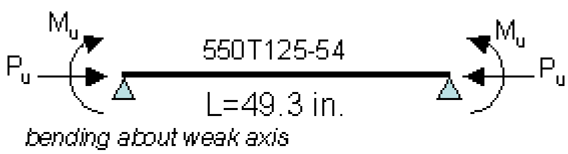
Warning: This example is provided to the reader so that a better understanding of the future direction that the Direct Strength Method is anticipated to take can be appreciated. Research in this area is currently active and changes may occur in the future. Conventional use of the beam-column interaction diagrams as shown in Example 8.3-5 can be overly conservative but is most closely consistent with the current *Specification*. Potential benefits of direct analysis of the cross-section for beam-columns are numerous and significant. This method has not been adopted by the *Specification*.

Given:
a. Steel: $F_y = 33$ ksi
b. Section SSMA Track 550T125-54 as shown to the right
c. Finite strip analysis results (Section 3.2.3)
d. Conventional example problem results as shown in Section 8.3 Design Examples 8.3-1 through 8.3-6.

Required
1. Beam-colum strength under P_u=2.8 kips, M_u=0.32 kip-in.



Consider the same beam-column as of Example 8.3-6.



First, consider the required strength (i.e., the demands):

Required strength (demands)

Axial

$P_u = 2.8\text{kip}$

$P_y = 14.91\text{kip}$

Bending

$\frac{C_m \cdot M_u}{\alpha} = 0.59\text{kip}\cdot\text{in}$ see Example 8.3-6 for C_m and α for details.

$M_y = 1.69\text{kip}\cdot\text{in}$

Characterizing the required strength (demand) more generally. Consider the typical interaction diagram as defining x and y coordinates, where $x=M_u/M_y$ and $y=P_u/P_y$. Now ask, how far away from the origin is the demand? Use β for this quantity.

$$x := \frac{C_m \cdot M_u}{\alpha \cdot M_y} \qquad y := \frac{P_u}{P_y} \qquad \beta_u := \sqrt{x^2 + y^2} \qquad \beta_u = 0.39$$

For this combination of P and M, how far can P and M be increased (together) before the first fiber yields? The ratio, γ, of f_y/f_{max} where f_{max} is the maximum stress in the cross-section based on gross properties allows us to calculate this distance.

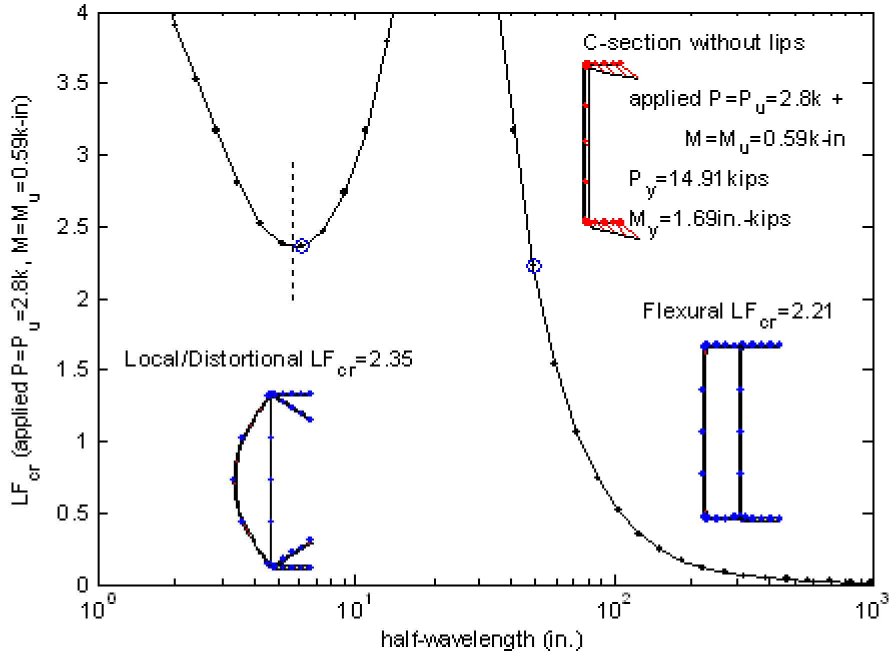
$$\gamma := \frac{33 \cdot \text{ksi}}{17.74\,\text{ksi}} \quad \text{where } 17.74\text{ksi} = P_u/A+(C_m M_u/\alpha)(c/I)$$

$\beta_y := \gamma \cdot \beta_u \qquad \beta_y = 0.73$ so we find $\frac{\beta_u}{\beta_y} = 0.54$ as a single parameter non-dimensional "demand" or "required strength"

(Continued) 6.3.1 direct beam-column analysis

Elastic buckling

Now perform finite strip analysis under this axial load and moment to find global, distortional, local buckling.



$\beta_{cre} := 2.21 \cdot \beta_u$ $\quad\quad$ $\beta_{cre} = 0.87$

$\beta_{crd} := 2.35 \cdot \beta_u$ $\quad\quad$ $\beta_{crd} = 0.93$

$\beta_{crl} := 2.35 \cdot \beta_u$ $\quad\quad$ $\beta_{crl} = 0.93$

The applied stress resulting from the application of the load and moment is shown in the upper right of the diagram. Note, M_u applied is the amplified, or approximate 2nd order required bending moment (demand). The local and global elastic buckling values under this applied stress are given to the left.

This is quite a lot to take in, but we may compare these results with the earlier pure beam and column results to try to maintain a sense of the magnitudes.

Global, compare beam-column direct $\quad 2.21 \cdot P_u = 6.18\text{kip}$ $\quad$ with comp. only $\quad P_{cre} = 6.08\text{kip}$

$2.21 \cdot M_u = 0.7\text{kip} \cdot \text{in}$ $\quad$ with bending only $\quad M_{cre} = 5.48\text{kip} \cdot \text{in}$

We see that the bending actually improves the global elastic buckling <u>axial</u> behavior.

Local, compare beam-column direct $\quad 2.35 \cdot P_u = 6.57\text{kip}$ $\quad$ with comp. only $\quad P_{crl} = 5.52\text{kip}$

$2.35 \cdot M_u = 0.74\text{kip} \cdot \text{in}$ $\quad$ with bending only $\quad M_{crl} = 4.26\text{kip} \cdot \text{in}$

Again, we see that bending actually improves the local elastic buckling <u>axial</u> behavior, in this section putting more compression on the lips is beneficial (to a point) because local buckling is initiated by the slender web.

(Continued) 6.3.1 direct beam-column analysis

STRENGTH (CAPACITY)

Final equations for global, local, and distortional buckling of beam-columns have not been determined, research is underway. However, the strength equations for columns are equal to or more conservative than the equations for beams *in all cases*. Therefore, use of the column expressions is a conservative choice at this juncture.

Global buckling check

$\lambda_c := \sqrt{\frac{\beta_y}{\beta_{cre}}}$ $\lambda_c = 0.89$ note, β replaces P, but otherwise the expressions are unmodified.

$$\beta_{ne} := \begin{cases} 0.658^{\left(\lambda_c^2\right)} \cdot \beta_y & \text{if } \lambda_c \le 1.5 \\ \frac{.877}{\lambda_c^2} \cdot \beta_y & \text{if } \lambda_c > 1.5 \end{cases}$$

$\beta_{ne} = 0.53$ (remember β_{demand} =0.39 and β_{yield} =0.73 for comparison)

Local buckling check

$\lambda_l := \sqrt{\frac{\beta_{ne}}{\beta_{crl}}}$ $\lambda_l = 0.777$

$$\beta_{nl} := \begin{cases} \beta_{ne} & \text{if } \lambda_l \le 0.776 \\ \left[\left[1 - 0.15 \cdot \left(\frac{\beta_{crl}}{\beta_{ne}}\right)^{0.4}\right]\left(\frac{\beta_{crl}}{\beta_{ne}}\right)^{0.4} \cdot \beta_{ne}\right] & \text{if } \lambda_l > 0.776 \end{cases}$$

$\beta_{nl} = 0.53$ We find essentially no local reduction! a significant change from Example 8.3-4.

Distortional buckling check

$\lambda_d := \sqrt{\frac{\beta_y}{\beta_{crd}}}$ $\lambda_d = 0.92$

$$\beta_{nd} := \begin{cases} \beta_y & \text{if } \lambda_d \le 0.561 \\ \left[\left[1 - 0.25 \cdot \left(\frac{\beta_{crd}}{\beta_y}\right)^{0.6}\right]\left(\frac{\beta_{crd}}{\beta_y}\right)^{0.6} \cdot \beta_y\right] & \text{if } \lambda_d > 0.561 \end{cases}$$

$\beta_{nd} = 0.59$ As described above inclusion of this check is conservative, but likely unnecessary. Note at this length DB does not control.

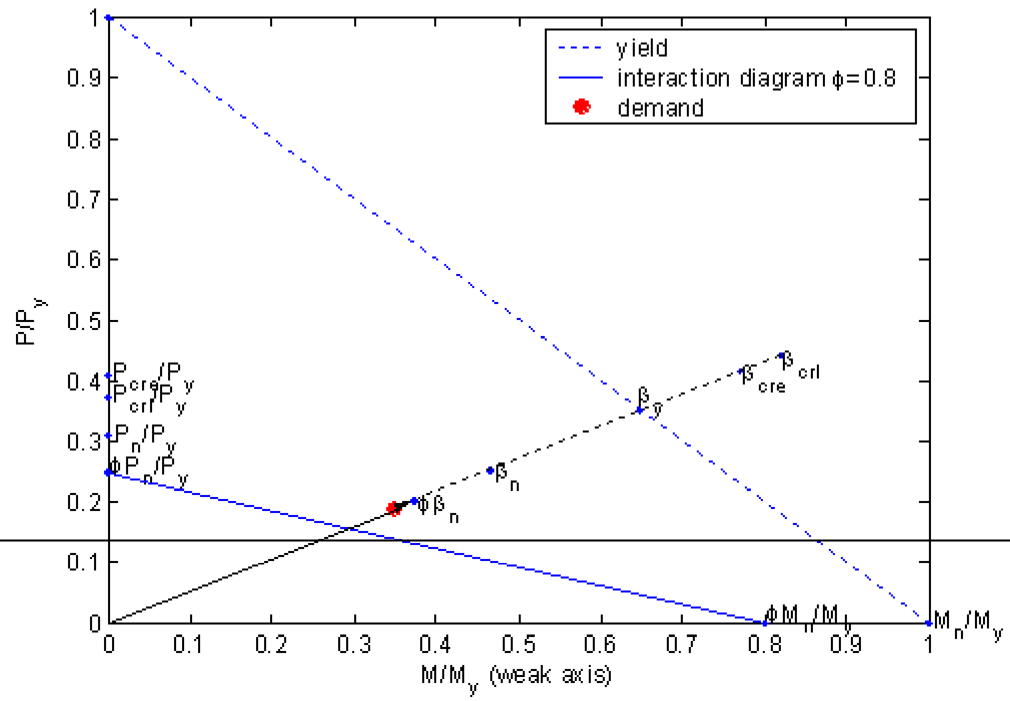
$\beta_n := \min\left(\left(\beta_{ne}\ \ \beta_{nl}\ \ \beta_{nd}\right)\right)$ $\beta_n = 0.53$

LRFD: $\phi_{bc} := 0.8$ $\phi_{bc} \cdot \beta_n = 0.42$ which is greater than the demand of 0.39, therefore the section is predicted to be **OK**

(Continued) 8.3-7 direct beam-column analysis

DISCUSSION

A visual interpretation of this analysis as compared to the traditional interaction diagram is embodied in the following figure.



Note M_{cre}/M_y and M_{crl}/M_y are >> 1 and therefore are not shown on the figure.

Beam: beam analysis results are on the horizontal axis, not shown is that the beam M_{cre}/M_Y and M_{crl}/M_Y are >>1. Note ϕM_n=0.8My is determined in Example 8.3-5.

Column: column analysis results are on the vertical axis. The plotted points are all determined in Example 8.3-4 and represent the pure column strength.

Interaction Diagram: The AISI *Specification* assumes a linear interaction diagram between the column and the beam strength as shown in the figure.

Direct Analysis: The line emenating from the origin represents a particular ratio of P to M demand, in this case $P=P_u$, $M=C_mM_u/\alpha$. The distances along this lines are the β values. Note, in this picture that the improvements in β_{cre} and β_{crl} are clear, they do not trend starkly lower due to the bending moment (quite the opposite). From an elastic stability standpoint the addition of this bending moment is beneficial. This helps explain why the DSM direct analysis provides a different solution from the interaction diagram, as detailed in Example 8.3-5.

The strength $\phi\beta_n$ could be determined for all ratios of P and M, and a new interaction diagram would then be generated. Correctly generated, this diagram would have the same endpoints as the traditional beam-column interaction equation, but the shape would be cross-section dependent.

7 Product development

7.1 Cross-section optimization

The examples of Chapters 3 and 8 provide some preliminary ideas on the optimization of cold-formed steel cross-sections. The given ideas are preliminary, and are not intended to suggest globally improved cross-sections. Nor are formal methods used for creating these improved cross-sections. The Direct Strength Method provides a tool that allows improvements in the cross-section to be examined by analysis, without testing, and without the inherent limitations of the existing provisions which focus on a limited number of cross-section types.

In most situations, the addition of small longitudinal stiffeners can greatly enhance the local buckling strength of cold-formed steel cross-sections. However, distortional buckling and global buckling are largely unaffected by such small changes. Thus, more creative enhancements may be pursued to increase distortional and/or global bucking strength.

Optimal designs predicted by the Direct Strength Method are known to be different from those predicted by simply manipulating the main *Specification* equations. One specific example of this: edge stiffeners are encouraged to be longer in DSM than in the main *Specification*.

Optimal designs also depend on the resistance (or safety) factor. Since, pre-qualified geometries employ higher ϕ factors (and lower Ω factors) than other geometries, which must follow rational analysis. Sections 7.3 and 7.4 of this Guide discuss this in greater detail and provide some basic advice on how to begin the process of determining the appropriate ϕ or Ω factor for cross-sections that are not pre-qualified.

Final optimization will have as much, if not more to do with manufacturing, constructability, and other practical matters; however, DSM provides a way to quantitatively focus on the strength improvements available to cold-formed steel designers/manufacturers. Cold-formed steel is a versatile, easily formed material – it is one objective of this Guide to help manufacturers take better advantage of the potential in cold-formed steel for creating optimal cross-section shapes.

7.2 Developing span and load tables

The solutions for beam charts and column charts discussed in Chapters 4 and 5 respectively, and demonstrated in Examples 8.13 and 8.14 respectively, provide the basis for developing span and load tables as typically generated by manufacturers.

Automation of the process of developing span and load tables is only possible once the elastic buckling values have been determined. Generally, determination of $P_{cr\ell}$, P_{crd}, P_{cre} and/or $M_{cr\ell}$, M_{crd}, and M_{cre} requires some manual interaction with an FSM program, etc. Once these values are established, and following the examples (8.13 and 8.14), their variation with unbraced length established, then developing the necessary tables is straightforward. Special care must be taken with resistance or safety factors (ϕ or Ω) since the DSM values may be different from the main *Specification*.

7.3 Rational analysis vs. Chapter F testing

For a developed cross-section not covered by the main *Specification* provisions two basic avenues exist for strength prediction, as outlined in the main *Specification* A1.1(b):

(a) determine the strength by testing and find ϕ via Chapter F of the *Specification*, or

(b) determine the strength by rational analysis and use the blanket ϕ provided.

The resistance factor (ϕ) for these two basic choices is shown in Figure 40.

Members		
USA and Mexico		Canada
Ω (ASD)	ϕ (LRFD)	ϕ (LSD)
2.00	0.80	0.75

(a) Rational analysis resistance factor

$$\phi = C_\phi (M_m F_m P_m) e^{-\beta_o \sqrt{V_M^2 + V_F^2 + C_P V_P^2 + V_Q^2}}$$

(b) resistance factor based on Chapter F

Figure 40 Resistance factor determination for a new cross-section

The rational analysis ϕ factor of 0.8 is lower (more conservative) than the typical ϕ factors for beams (0.9 to 0.95) and columns (0.85). However, when compared with a method relying solely on testing (Chapter F), there is an incentive towards rational analysis methods.

If rational analysis is **<u>not</u>** employed, and instead the strength prediction of a member is based solely on testing, then the provisions of Chapter F of the main *Specification* apply. In particular, the expression of Figure 40(b) governs. The number of tests required to generate a more favorable resistance factor than the rational analysis procedure can be considerable, and depends on the scatter (coefficient of variation) of the test results. Given the rational analysis ϕ value of 0.8, Figure 41 shows that for typical test scatter (V_P=15%) at least 11 tests must be performed before a pure test method generates a ϕ > 0.80. If the scatter is less, fewer tests are needed. To generate ϕ factors as high as the pre-qualified Direct Strength Method expressions the scatter must be low *and* a significant number of tests must be performed.

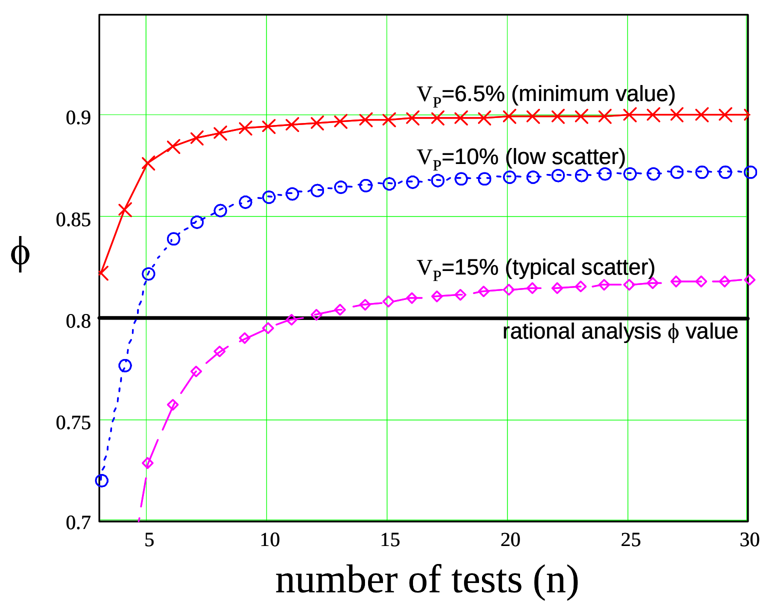


Figure 41 Relationship between number of tests and ϕ for Chapter F method

Given that testing is generally costly, and scatter is inherent in cold-formed steel systems (a low V_P is hard to achieve) the rational analysis method ϕ (resistance) factor is a definite incentive.

7.4 New pre-qualified members and extending the bounds of a pre-qualified member

No formal method yet exists in the *Specification* for extending the Direct Strength Method to additional (new) pre-qualified members, or extending the bounds for existing pre-qualified members. Currently, to extend the bounds or add an additional cross-section, a formal ballot vote of the AISI Committee on Specifications is required.

As shown in the example problems of Chapter 8 the pre-qualified ϕ factors make a significant difference in determining the design strength, and hence optimum member design. If logical, it would be desirable for new cross-sections or cross-sections that slightly violate existing pre-qualified bounds to employ the pre-qualified resistance factors. To justify such use it must be ensured that the reliability of the design strength is not compromised. This requires one to take a closer look at how the reliability of the Direct Strength Method was determined.

Table 5 is provided to help the user of this Guide to better understand the statistics underlying the reliability of the Direct Strength Method. This table shows the number of samples (n), the mean test-to-predicted ratio for DSM (P_m) and the coefficient of variation of the test-to-predicted ratio (V_P) for each of the cross-section types identified in Tables 1.1.1-1 and 1.1.1-2 in DSM. These statistics, in conjunction with the method outlined in Chapter F, provide the evidence that the pre-qualified members can meet (or exceed) the target reliability, and thus may use higher ϕ factors than the Section A1.1(b) rational analysis values. Thus ϕ for pre-qualified beams is 0.9 and ϕ for pre-qualified columns is 0.85.

Table 5 Summary Statistics for DSM Development

	n	P_m	V_P
Beams			
C-sections	185	1.10	0.11
C-sections with web stiffeners	42	1.12	0.07
Z-sections	48	1.13	0.13
Hat sections	186	1.10	0.15
Trapezoidal sections	98	1.01	0.13
ALL BEAMS	**559**	**1.09**	**0.12**
Columns			
C-sections	114	1.01	0.15
C-sections with web stiffeners[1]	29	0.88	0.14
Z-sections	85	0.96	0.13
Rack sections	17	1.02	0.05
Hat sections	4	0.98	0.02
ALL COLUMNS	**249**	**0.98**	**0.14**

(1) Thomasson's (1978) tests contribute to the low P_m, more recent tests by Kwon and Hancock (1992) showed much better agreement.

New cross-section: consider the case where one is interested in pre-qualifying an entirely new cross-section. Do three or more tests. Generate a ϕ using Chapter F, **but** use the test-to-predicted ratio with DSM as the prediction to generate the professional factor P_m, and use the coefficient of variation of the test-to-predicted ratio for V_P. (In a method relying solely on testing P_m is 1.0 and V_P is the scatter in the test results, here P_m is the mean accuracy of the predicting method, and V_P is the scatter in the predicting method). If the ϕ produced is greater than or equal to a DSM pre-qualified cross-section (ϕ=0.9 for beams, ϕ=0.85 for columns) this is *strong* evidence that the cross-section should be pre-qualified. The proposed bounds for the new cross-section would be the bounds of the testing.

Pilot tests for a new cross-section: consider the case where pilot tests on a new cross-section are being considered but a large battery of tests cannot be performed at the time. Assuming only a small number of tests can be performed the following procedure is suggested. Perform at least three tests and determine the average test-to-predicted ratio for DSM (P_m^*). For the coefficient of variation for the method (V_P) assume the worst V_P observed in Table 5, 15%. Set the correction for sample size, C_p to 1.0. Estimating the mean with a small number of tests is more reliable than estimating V_P, so this method attempts to assure that the reliability (in essence, ϕ) is met, by assuming that the variation (V_P) is the most conservative V_P observed to date. Now, determine the resistance factor ϕ, via Chapter F of the main *Specification*. If the ϕ produced is greater than or equal to a DSM pre-qualified cross-section (ϕ=0.9 for beams, ϕ=0.85 for columns) this is evidence that the cross-section should be pre-qualified. A lower ϕ is not cause for immediate rejection, but does suggest more tests or a revised strength prediction equation may be needed.

Extend the bounds: consider the case where engineering judgment makes it clear that a cross-section fits in one of the pre-qualified categories, but one or more of the geometric bounds are violated. If a large battery of tests can be performed then the procedure could follow that of a new cross-section as described in the preceding paragraphs. Assuming only a small number of tests can be performed the following procedure is suggested. Perform at least three tests and determine the average test-to-predicted ratio for DSM (P_m^*). If the P_m^* of the three tests is equal to or greater than the comparable pre-qualified category (Table 5), then the bounds should likely be extended. Estimating the mean with a small number of tests is more reliable than estimating V_P, so this method assures that the reliability (in essence, ϕ) is met, by assuming that the variation (V_P) in the newly tested cross-sections is equal to that of the underlying category. In no way does a lower P_m^* preclude that the cross-section should be pre-qualified, but additional testing will likely be required.

Notes: The preceding procedures do not guarantee the cross-section will be pre-qualified; they are an attempt to provide manufacturers with the best current advice. Currently, testing and calculation evidence would need to be taken to the AISI Committee on Specifications in the form of a ballot for consideration to extend pre-qualified cross-sections. All test results would need to be available to the public. All quantities should be measured, not nominal. The thickness should be the measured base metal thickness, the yield stress should be based on tensile coupons from the as-formed cross-section, and the dimensions should be based on direct measurement. For ASD Ω can be calculated from ϕ via main *Specification* Equation F1.2-2.

8 Design examples

The design example presented here include those cross-sections employed in the AISI (2002) *Design Manual* plus additional cross-sections selected to highlight the use of the Direct Strength Method for more complicated and optimized members. The following cross-sections are considered:

- 8.1 C-section with lips,
- 8.2 C-section with lips *modified*,
- 8.3 C-section without lips (track section),
- 8.4 C-section without lips (track section) *modified*,
- 8.5 Z-section with lips,
- 8.6 Z-section with lips *modified*,
- 8.7 Equal leg angle with lips,
- 8.8 Equal leg angle,
- 8.9 Hat section,
- 8.10 Wall panel section,
- 8.11 Rack post section, and a
- 8.12 Sigma section.

The relationship between the AISI (2002) *Design Manual* examples and the Design Examples of this Guide are provided for beams in Table 2 of Section 4.5, for columns in Table 3 of Section 5.3, and for beam-columns in Table 4 of Section 6.2. Numerical comparison of the predicted strength between DSM and the main *Specification* is provided in Section 8.15 of this Guide.

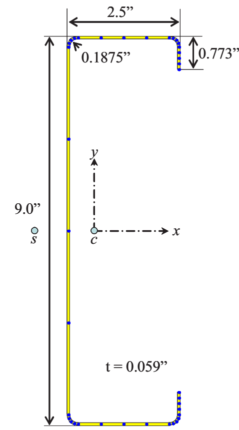
For each cross-section the design flexural strength for a fully braced beam and the design compressive strength for a continuously braced column are provided. For most cross-sections additional design strength values for discrete bracing in bending and compression are provided. For the C-section with lips, C-section without lips, Equal leg angle, and the hat section, beam-column examples are provided.

The Design Examples were prepared using MathCAD. For readers unfamiliar with the notations used in this software please see the Quick Start guide on page iv of this Guide. Also, to better understand how MathCAD works and to enable the reader to check intermediate calculation values Design Example 8.1.1 is completed in standard MathCAD format *and* in an extended format. In the extended format all values are explicitly substituted into the equations and all intermediate calculation results are provided to the reader.

The Design Examples given in Sections 8.13 and 8.14 provide the complete solution for developing beam charts and column charts, respectively. The cross-sections employed in these examples are the C-section with lips and the C-section with lips *modified*.

8.1 C-section with lips

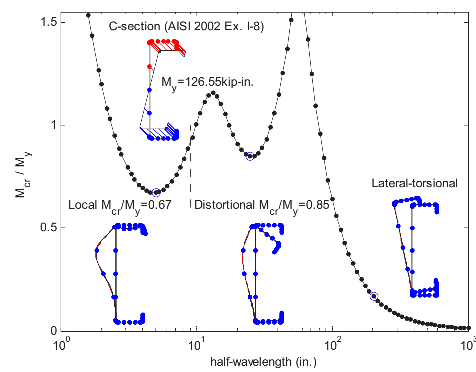
Given:
a. Steel: $F_y = 55$ ksi
b. Section 9CS2.5x059 as shown to the right
c. Finite strip analysis results (Section 3.2.1)
Required:
1. Flexural strength for a fully braced member
2. Flexural strength for L=56.2 in. (AISI 2002 Example II-1)
3. Effective moment of inertia
4. Compressive strength for a fully braced member
5. Compressive strength at F_n=37.25 ksi (AISI 2002 Example III-1)
6. Beam-column design strength (AISI 2002 Example III-1)



8.1-1 Flexural strength for a fully braced member (AISI 2002 Example I-8)

Determination of the nominal flexural strength for a fully braced member is equivalent to determining the effective section modulus at yield in the main *Specification*. See AISI (2002) example I-8.

Finite strip analysis of 9CS2.5x059 in pure bending as summarized in Example 3.2.1



Inputs from the finite strip analysis include:

$M_y := 126.55 \cdot \text{kip} \cdot \text{in}$

$M_{crl} := 0.67 \cdot M_y$ $\qquad M_{crl} = 85\,\text{kip} \cdot \text{in}$

$M_{crd} := 0.85 \cdot M_y$ $\qquad M_{crd} = 108\,\text{kip} \cdot \text{in}$

Per DSM 1.2.2, M_n is the minimum of M_{ne}, $M_{n\ell}$, M_{nd}. For a fully braced member lateral-torsional buckling will not occur and thus $M_{ne} = M_y$, $M_{n\ell}$ and M_{nd} must still be checked.

$M_{ne} := M_y$ $\qquad M_{ne} = 127\,\text{kip} \cdot \text{in}$ $\qquad$ (fully braced)

Local buckling check per DSM 1.2.2.2

$$\lambda_l := \sqrt{\frac{M_{ne}}{M_{crl}}} \qquad \lambda_l = 1.22 \qquad \text{(subscript "l" = "}\ell\text{")} \qquad \text{(Eq. 1.2.2-7)}$$

$$M_{nl} := \begin{cases} M_{ne} & \text{if } \lambda_l \le 0.776 \\ \left[1 - 0.15 \cdot \left(\frac{M_{crl}}{M_{ne}}\right)^{0.4}\right]\left(\frac{M_{crl}}{M_{ne}}\right)^{0.4} \cdot M_{ne} & \text{if } \lambda_l > 0.776 \end{cases} \qquad \begin{matrix}\text{(Eq. 1.2.2-5)} \\ \text{(Eq. 1.2.2-6)}\end{matrix}$$

$M_{nl} = 94\,\text{kip} \cdot \text{in}$

(Continued) 8.1-1 Flexural strength for a fully braced member

Distortional buckling check per DSM 1.2.2.3

$$\lambda_d := \sqrt{\frac{M_y}{M_{crd}}} \qquad \lambda_d = 1.08 \qquad \text{(Eq. 1.2.2-10)}$$

$$M_{nd} := \begin{cases} M_y \ \text{ if } \lambda_d \le 0.673 & \text{(Eq. 1.2.2-8)} \\ \left[1 - 0.22\cdot\left(\frac{M_{crd}}{M_y}\right)^{0.5}\right]\left(\frac{M_{crd}}{M_y}\right)^{0.5}\cdot M_y \ \text{ if } \lambda_d > 0.673 & \text{(Eq. 1.2.2-9)} \end{cases}$$

$M_{nd} = 93\,\text{kip}\cdot\text{in}$

Predicted flexural strength per DSM 1.3

$M_n := \min\left(\left(M_{ne} \;\; M_{nl} \;\; M_{nd}\right)\right) \qquad M_n = 93\,\text{kip}\cdot\text{in}$

The geometry of this section falls within the "pre-qualified" beams of DSM 1.1.1.2 and the higher ϕ and lower Ω of DSM Section 1.2.2 may therefore be used.

LRFD: $\phi_b := 0.9 \qquad \phi_b\cdot M_n = 84\,\text{kip}\cdot\text{in}$

ASD: $\Omega_b := 1.67 \qquad \frac{M_n}{\Omega_b} = 56\,\text{kip}\cdot\text{in}$

Note on uplift

Part 4 of the AISI (2002) *Design Manual* Example II-1 calculates the nominal flexural strength in uplift of this cross-section. The formula is RS_eF_y, where S_eF_y is the fully braced flexural strength and R is an empirical factor from main *Specification* C3.1.3. If the conditions of C3.1.3 are met, flexural strength in uplift can be found using this simple manner. For this example R=0.6.

uplift: $R := 0.6 \qquad R\cdot M_n = 55.81\,\text{kip}\cdot\text{in}$ nominal flexural strength in uplift

The calibration of R factors was performed to the main *Specification*. The main *Specification* does not include an explicit check for distortional buckling. Therefore, if distortional buckling (M_{nd}) controls the strength the uplift prediction using R may be excessively conservative. Here M_{nd}~$M_{n\ell}$ and it is expected that the prediction is reasonably accurate.

8.1-1 REWORKED WITH EXPLICIT SUBSITITION OF INTERMEDIATE VALUES

8.1-1 Flexural strength for a fully braced member (AISI 2002 Example I-8)

Consider the solution to 8.1-1 again, but now with all intermediate values shown so that the reader may be sure of how the expressions are employed, and check intermediate values.

Inputs from the finite strip analysis:

Finite strip analysis of 9CS2.5x059 in pure bending as summarized in Section 3.2.1

$M_y := 126.55 \cdot kip \cdot in$

Elastic critical local buckling

$M_{crl} := 0.67 \cdot M_y$ $\quad M_{crl} := 0.67 \cdot 126.55 \cdot kip \cdot in$ $\quad M_{crl} = 85\, kip \cdot in$

Elastic critical distortional buckling

$M_{crd} := 0.85 \cdot M_y$ $\quad M_{crd} := 0.85 \cdot 126.55 \cdot kip \cdot in$ $\quad M_{crd} = 108\, kip \cdot in$

Lateral-torsional buckling check per DSM 1.2.2.1

Per DSM 1.2.2, M_n is the minimum of M_{ne}, $M_{n\ell}$, M_{nd}. For a fully braced member lateral-torsional buckling will not occur and thus $M_{ne} = M_y$, $M_{n\ell}$ and M_{nd} must still be checked.

$M_{ne} := M_y$ $\quad M_{ne} = 127\, kip \cdot in$ $\quad$ (fully braced)

Local buckling check per DSM 1.2.2.2

$$\lambda_l := \sqrt{\frac{M_{ne}}{M_{crl}}} \qquad \lambda_l := \sqrt{\frac{127 \cdot kip \cdot in}{85 \cdot kip \cdot in}} \qquad \lambda_l = 1.22 \qquad \text{(Eq. 1.2.2-7)}$$

$$M_{nl} := \begin{cases} M_{ne} & \text{if } \lambda_l \le 0.776 \\ \left[1 - 0.15 \cdot \left(\frac{M_{crl}}{M_{ne}}\right)^{0.4}\right]\left(\frac{M_{crl}}{M_{ne}}\right)^{0.4} \cdot M_{ne} & \text{if } \lambda_l > 0.776 \end{cases} \qquad \begin{array}{r}\text{(Eq. 1.2.2-5)}\\ \text{(Eq. 1.2.2-6)}\end{array}$$

$$M_{nl} := \begin{cases} 127 \cdot kip \cdot in & \text{if } \lambda_l \le 0.776 \\ \left[1 - 0.15 \cdot \left(\frac{85}{127}\right)^{0.4}\right]\left(\frac{85}{127}\right)^{0.4} \cdot 127 \cdot kip \cdot in & \text{if } \lambda_l > 0.776 \end{cases}$$

$\lambda_l > 0.776$, therefore $\quad M_{nl} := (.87).85 \cdot 127 \cdot kip \cdot in$ $\quad M_{nl} = 94\, kip \cdot in$

(continued)

8.1-1 REWORKED WITH EXPLICIT SUBSITITION OF INTERMEDIATE VALUES

Distortional buckling check per DSM 1.2.2.3

$\lambda_d := \sqrt{\frac{M_y}{M_{crd}}}$ $\quad \lambda_d := \sqrt{\frac{126.55 \cdot kip \cdot in}{108 \cdot kip \cdot in}}$ $\quad \lambda_d = 1.08$ (Eq. 1.2.2-10)

$$M_{nd} := \begin{vmatrix} M_y \text{ if } \lambda_d \le 0.673 \\ \left[1 - 0.22 \cdot \left(\frac{M_{crd}}{M_y}\right)^{0.5}\right]\left(\frac{M_{crd}}{M_y}\right)^{0.5} \cdot M_y \text{ if } \lambda_d > 0.673 \end{vmatrix}$$

(Eq. 1.2.2-8)
(Eq. 1.2.2-9)

$$M_{nd} := \begin{vmatrix} 126.55 \cdot kip \cdot in \text{ if } \lambda_d \le 0.673 \\ \left[1 - 0.22 \cdot \left(\frac{108}{126.55}\right)^{0.5}\right]\left(\frac{108}{126.55}\right)^{0.5} \cdot 126.55 \cdot kip \cdot in \text{ if } \lambda_d > 0.673 \end{vmatrix}$$

$\lambda_d > 0.673$, therefore $\quad M_{nd} := (.80).92 \cdot 126.55 \cdot kip \cdot in \quad M_{nd} = 93\,kip \cdot in$

Predicted flexural strength per DSM 1.3

$M_n := \min((M_{ne}\ \ M_{nl}\ \ M_{nd}))$

$M_n := \min((127 \cdot kip \cdot in\ \ 94 \cdot kip \cdot in\ \ 93 \cdot kip \cdot in))$

$M_n = 93\,kip \cdot in$

The geometry of this section falls within the "pre-qualified" beams of DSM 1.1.1.2 and the higher ϕ and lower Ω of DSM Section 1.2.2 may therefore be used.

LRFD: $\quad \phi_b := 0.9 \quad \phi_b \cdot M_n = 84\,kip \cdot in$ $\quad$ flexural design strength

ASD: $\quad \Omega_b := 1.67 \quad \frac{M_n}{\Omega_b} = 56\,kip \cdot in$ $\quad$ flexural allowable strength

8.1-2 Flexural strength for L=56.2 in. (AISI 2002 Example II-1)

AISI (2002) Example II-1 provides a complete calculation for a four span continuous beam. The following calculation provides an alternative means to calculate the flexural design strength of one of the spans. Namely, an interior span where L_y=L_t=56.2 in., and C_b = 1.67 (conservatively assumed as a linear moment diagram between the inflection point and the support).

Inputs from the finite strip analysis include:

Local and distortional are unchanged from Example 8.1-1

Global (lateral-torsional) buckling may be found directly from the finite strip analysis plot

$M_{cre} := 1.73 \cdot M_y$ for C_b=1 at L_y=L_t=56.2 in. $M_{cre} = 218.93\,\text{kip}\cdot\text{in}$

Lateral-torsional buckling check per DSM 1.2.2.1

per AISI (2002) Example II-1 $C_b := 1.67$

$M_{cre} := C_b \cdot M_{cre}$ $\frac{M_{cre}}{M_y} = 2.89$ per upperbounds of Section 2.2 M_{ne}=M_y!

more formally,

$$M_{ne} := \begin{cases} M_{cre} & \text{if } M_{cre} < 0.56 \cdot M_y \quad \text{(Eq. 1.2.2-1)} \\ \frac{10}{9} \cdot M_y \cdot \left(1 - \frac{10 \cdot M_y}{36 \cdot M_{cre}}\right) & \text{if } 2.78 \cdot M_y \ge M_{cre} \ge 0.56 \cdot M_y \quad \text{(Eq. 1.2.2-2)} \\ M_y & \text{if } M_{cre} > 2.78 \cdot M_y \quad \text{(Eq. 1.2.2-3)} \end{cases}$$

$M_{ne} = 126.55\,\text{kip}\cdot\text{in}$

Local buckling check per DSM 1.2.2.2

unchanged from Example 8.1-1 $M_{nl} = 94\,\text{kip}\cdot\text{in}$

Distortional buckling check per DSM 1.2.2.3

unchanged from Example 8.1-1 $M_{nd} = 93\,\text{kip}\cdot\text{in}$

Predicted flexural strength per DSM 1.3

$M_n := \min\left(\left(M_{ne}\ \ M_{nl}\ \ M_{nd}\right)\right)$ $M_n = 93\,\text{kip}\cdot\text{in}$

The geometry of this section falls within the "pre-qualified" beams of DSM Section 1.1.1.2 and the higher ϕ and lower Ω of DSM Section 1.2.2 may therefore be used.

LRFD: $\phi_b := 0.9$ $\phi_b \cdot M_n = 84\,\text{kip}\cdot\text{in}$ Note, that ϕ_b from DSM Section 1.2.2 is 0.90, not 0.95 as in the main *Specification*.

ASD: $\Omega_b := 1.67$ $\frac{M_n}{\Omega_b} = 56\,\text{kip}\cdot\text{in}$

Further notes on AISI (2002) Design Example II-1: Design checks for shear could follow the nomenclature of Section 4.4.1 of this Guide. Design checks for web crippling are unmodified from the AISI (2002) example. In the design checks for combined bending and shear, and combined bending and crippling ϕM_{nxo} could be replaced by the results for ϕM_n from Example 8.1-1.

8.1-3 Effective Moment of Inertia (AISI 2002 Example I-8)

The Direct Strength Method prescribes that the reduction in the bending stiffness can be determined by finding the ratio of the nominal flexural strength at service loads (M_d) to the applied loads, M.

$$I_{eff} := \frac{M_d}{M} \cdot I_g \quad < I_g \qquad \text{(Eq. 1.1.3-1)}$$

M: Consider a service load at 60% of nominal strength (where M_n = 93 kip-in. from Example 8.1-1)

$M := 0.6 \cdot M_n \qquad M = 55.88\ \text{kip} \cdot \text{in}$

$\mathbf{M_d}$: local/distortional strength reduction at required (demand) M

M_{crl} and M_{crd} from finite strip analysis remain the same as given in Examples 8.1-1 and 8.1-2

Determine M_{de} for global buckling (per DSM 1.2.2.1) (replace M_y with M as stated in Eq. 1.1.3-1)

For a fully braced section, or for the scenario of Example 8.1-2, M_{cre}>2.78M therefore, per Eq. 1.2.2-3 no reduction will occur due to global buckling.

$M_{de} := M \qquad$ (Eq. 1.2.2-3)

$M_{de} = 55.88\ \text{kip} \cdot \text{in}$ note subscript "n" is replaced with "d" to denote **d**eflection calculation, so M_{ne} becomes M_{de}, $M_{n\ell}$ becomes $M_{d\ell}$ and M_{nd} becomes M_{dd}.

Determine $M_{d\ell}$ for local buckling (per DSM 1.2.2.2)

$$\lambda_l := \sqrt{\frac{M_{de}}{M_{crl}}} \qquad \lambda_l = 0.81 \qquad \text{Note } M_{ne} \text{ replaced with } M_{de} \qquad \text{(Eq. 1.2.2-7)}$$

$$M_{dl} := \begin{cases} M_{de} & \text{if } \lambda_l \le 0.776 \\ \left[1 - 0.15 \cdot \left(\frac{M_{crl}}{M_{de}}\right)^{0.4}\right]\left(\frac{M_{crl}}{M_{de}}\right)^{0.4} \cdot M_{de} & \text{if } \lambda_l > 0.776 \end{cases} \qquad M_{dl} = 54.3\ \text{kip} \cdot \text{in}$$

(Eq. 1.2.2-5)
(Eq. 1.2.2-6)

Determine M_{dd} for distortional buckling (per DSM 1.2.2.3)

replace M_y with M as stated in Eq. 1.1.3-1

$$\lambda_d := \sqrt{\frac{M}{M_{crd}}} \qquad \lambda_d = 0.72 \qquad \text{Note } M_y \text{ replaced with } M \qquad \text{(Eq. 1.2.2-10)}$$

$$M_{dd} := \begin{cases} M & \text{if } \lambda_d \le 0.673 \\ \left[1 - 0.22 \cdot \left(\frac{M_{crd}}{M}\right)^{0.5}\right]\left(\frac{M_{crd}}{M}\right)^{0.5} \cdot M & \text{if } \lambda_d > 0.673 \end{cases} \qquad M_{dd} = 53.9\ \text{kip} \cdot \text{in}$$

(Eq. 1.2.2-8)
(Eq. 1.2.2-9)

Final M_d for determining the reduced moment of inertia in Eq. 1.1.3-1

$M_d := \min((M_{de}\ \ M_{dd}\ \ M_{dl})) \qquad M_d = 53.9\ \text{kip} \cdot \text{in}$

$$I_{eff} := \frac{M_d}{M} \cdot I_g \quad < I_g \qquad \text{(Eq. 1.1.3-1)}$$

$I_{eff} = 9.93\ \text{in}^4 \qquad \frac{I_{eff}}{I_g} = 0.96$ At a demand of 60% of the nominal moment capacity the predicted stiffness is 96% of the gross moment of inertia.

(Continued) 8.1-3 Effective Moment of Inertia (AISI 2002 Example I-8)

I_{eff} at any required strength (moment demand) less than the nominal flexural strength

M: Consider required strengths (moment demands) up to the nominal flexural strength, find I_{eff}

$$M(\alpha) := \alpha \cdot M_n \qquad \text{where} \qquad \alpha := 0.01, 0.02 .. 1$$

M_d: local/distortional strength reduction at demand (required strength) M

Determine M_{de} for global buckling (per DSM 1.2.2.1)

Replace M_y with M in Eqs. 1.2.2-1 and -3 as stated in Eq. 1.1.3-1. If the section is fully braced then M_{cre} is > 2.78M, otherwise put M_{cre} for the actual unbraced length in Eqs. 1.2.2-1 and -3.

Assume the section is fully braced, therefore: $M_{de}(\alpha) := M(\alpha)$

Determine $M_{d\ell}$ for local buckling (per DSM 1.2.2.2)

$$\lambda_l(\alpha) := \sqrt{\frac{M_{de}(\alpha)}{M_{crl}}} \qquad \text{(Eq. 1.2.2-7)}$$

$$M_{dl}(\alpha) := \begin{cases} M_{de}(\alpha) & \text{if } \lambda_l(\alpha) \le 0.776 \\ \left[1 - 0.15\cdot\left(\frac{M_{crl}}{M_{de}(\alpha)}\right)^{0.4}\right]\left(\frac{M_{crl}}{M_{de}(\alpha)}\right)^{0.4}\cdot M_{de}(\alpha) & \text{if } \lambda_l(\alpha) > 0.776 \end{cases} \qquad \begin{matrix}\text{(Eq. 1.2.2-5)}\\ \text{(Eq. 1.2.2-6)}\end{matrix}$$

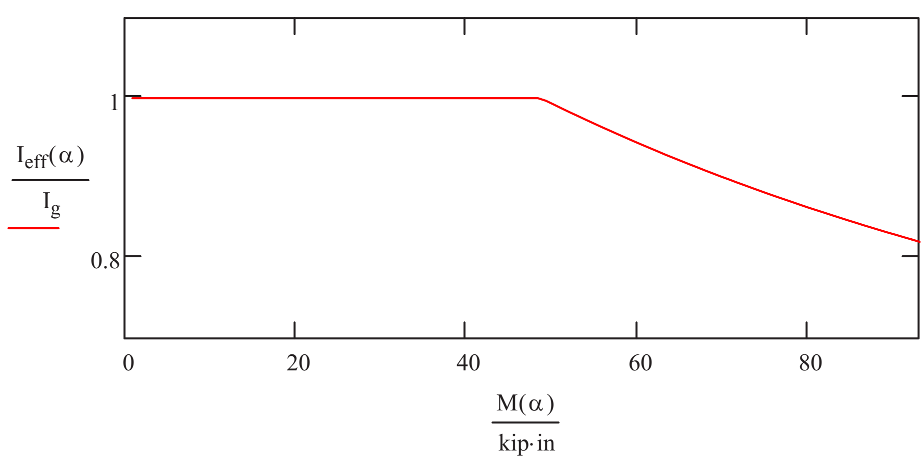
Determine M_{dd} for distortional buckling (per DSM 1.2.2.3)

$$\lambda_d(\alpha) := \sqrt{\frac{M(\alpha)}{M_{crd}}} \qquad \text{replace } M_y \text{ with M, as prescribed} \qquad \text{(Eq. 1.2.2-10)}$$

$$M_{dd}(\alpha) := \begin{cases} M(\alpha) & \text{if } \lambda_d(\alpha) \le 0.673 \\ \left[1 - 0.22\cdot\left(\frac{M_{crd}}{M(\alpha)}\right)^{0.5}\right]\left(\frac{M_{crd}}{M(\alpha)}\right)^{0.5}\cdot M(\alpha) & \text{if } \lambda_d(\alpha) > 0.673 \end{cases} \qquad \begin{matrix}\text{(Eq. 1.2.2-8)}\\ \text{(Eq. 1.2.2-9)}\end{matrix}$$

$$M_d(\alpha) := \min\left(\left(M_{de}(\alpha) \;\; M_{dd}(\alpha) \;\; M_{dl}(\alpha)\right)\right)$$

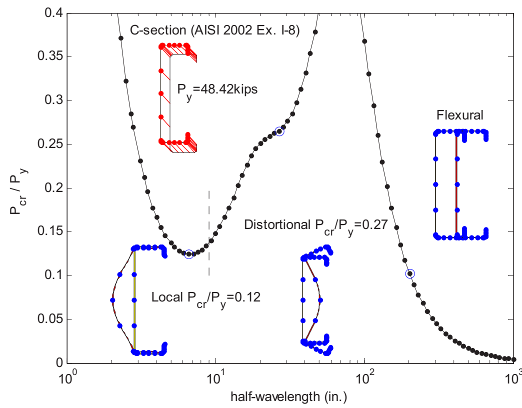
Effective moment of inertia $I_{eff}(\alpha) := I_g \cdot \frac{M_d(\alpha)}{M(\alpha)}$



moment demand (kip-in.)

8.1-4 Compressive strength for a fully braced column (AISI 2002 Example I-8)

Finite strip analysis of 9CS2.5x059 in pure compression is summarized in Example 3.2.1



Inputs from the finite strip analysis include:

$P_y := 48.42 \cdot kip$

$P_{crl} := 0.12 \cdot P_y$ $\quad P_{crl} = 5.8\,kip$

$P_{crd} := 0.27 \cdot P_y$ $\quad P_{crd} = 13.1\,kip$

Per DSM 1.2.1, P_n is the minimum of P_{ne}, $P_{n\ell}$, P_{nd}. If a column is continuously braced then global buckling P_{ne} is restricted and the squash load will develop if the section is compact.

$P_{ne} := P_y$ $\quad P_{ne} = 48.42\,kip$

Local buckling check per DSM 1.2.1.2

$$\lambda_l := \sqrt{\frac{P_{ne}}{P_{crl}}} \qquad \lambda_l = 2.89 \qquad \text{(subscript "l" = "}\ell\text{")} \qquad \text{(Eq. 1.2.1-7)}$$

$$P_{nl} := \begin{cases} P_{ne} & \text{if } \lambda_l \le 0.776 \\ \left[1 - 0.15 \cdot \left(\frac{P_{crl}}{P_{ne}}\right)^{0.4}\right]\left(\frac{P_{crl}}{P_{ne}}\right)^{0.4} \cdot P_{ne} & \text{if } \lambda_l > 0.776 \end{cases} \qquad \begin{matrix}\text{(Eq. 1.2.1-5)}\\ \text{(Eq. 1.2.1-6)}\end{matrix}$$

$P_{nl} = 19.4\,kip$ (note the significant post-buckling strength, $P_{cr\ell}$ is only 5.8 kips)

Distortional buckling check per DSM 1.2.1.3

$$\lambda_d := \sqrt{\frac{P_y}{P_{crd}}} \qquad \lambda_d = 1.92 \qquad \text{(Eq. 1.2.1-10)}$$

$$P_{nd} := \begin{cases} P_y & \text{if } \lambda_d \le 0.561 \\ \left[1 - 0.25 \cdot \left(\frac{P_{crd}}{P_y}\right)^{0.6}\right]\left(\frac{P_{crd}}{P_y}\right)^{0.6} \cdot P_y & \text{if } \lambda_d > 0.561 \end{cases} \qquad \begin{matrix}\text{(Eq. 1.2.1-8)}\\ \text{(Eq. 1.2.1-9)}\end{matrix}$$

$P_{nd} = 19.6\,kip$

Predicted compressive capacity per DSM 1.2

$P_n := \min\left(\left(P_{ne} \;\; P_{nl} \;\; P_{nd}\right)\right)$ $\qquad P_n = 19.4\,kip$

The geometry of this section falls within the "pre-qualified" columns of DSM Section 1.1.1.1 and the higher ϕ and lower Ω of DSM Section 1.2.1 may therefore be used.

LRFD: $\phi_c := 0.85 \quad \phi_c \cdot P_n = 16.5\,kip$ $\qquad$ ASD: $\Omega_c := 1.80 \quad \frac{P_n}{\Omega_c} = 10.8\,kip$

8.1-5 Compressive strength at F_n=37.25 ksi (AISI 2002 Example III-1)

In AISI (2002) Example III-1 this 9CS2.5x059 is examined as a 20 ft. long beam-column. The section is simply-supported at its ends, and fully braced against lateral and torsional buckling. The compressive design strength of that beam-column is the subject of this example.

Flexural, Torsional, or Torsional-Flexural check per DSM 1.2.1.1

The section is restricted so that it can only buckle about the strong axis. This restriction could be imposed on the finite strip model, as described in Section 3.4.6. However, this is more cumbersome than simply calculating the elastic buckling strength using closed-form formulas.

The strong axis elastic buckling stress may be found as detailed in AISI (2002) Example III-1, but involves nothing more than calculating $F_e=\pi^2E/(KL_x/r_x)^2$, for this section

$F_e := 59.12 \cdot ksi$ (AISI 2002 Example III-1)

$P_{cre} := A_g \cdot F_e$ $P_{cre} = 52.05\ kip$

$$\lambda_c := \sqrt{\frac{P_y}{P_{cre}}} \qquad \lambda_c = 0.96 \quad \text{(inelastic regime)} \qquad \text{(Eq. 1.2.1-1)}$$

$$P_{ne} := \begin{cases} 0.658^{\lambda_c^2} \cdot P_y & \text{if } \lambda_c \le 1.5 \quad \text{(Eq. 1.2.1-2)} \\ \dfrac{.877}{\lambda_c^2} \cdot P_y & \text{if } \lambda_c > 1.5 \quad \text{(Eq. 1.2.1-3)} \end{cases}$$

$P_{ne} = 32.8\ kip$

The stress associated with this load is $\frac{P_{ne}}{A_g} = 37.26\ ksi$ which is F_n of the main *Specification*.

As demonstrated in AISI (2002) Example I-8, in the main *Specification* the effective area (A_e) calculations for columns are performed at the stress that the long column can maintain, this stress known as F_n is calculated per Eq. C4-2 or C4-3. A similar procedure is performed in the Direct Strength Method where the local buckling slenderness and strength equations (DSM Eqs. 1.2.1-7, and 1.2.1-5,6) are calculated at the strength a long column can maintain, i.e., $P_{ne}=A_gF_n$

Local buckling check per DSM 1.2.1.2

$P_{crl} = 5.8\ kip$ (from Example 8.1-4)

$$\lambda_l := \sqrt{\frac{P_{ne}}{P_{crl}}} \qquad \lambda_l = 2.38 \qquad \text{(Eq. 1.2.1-7)}$$

(compare with λ_ℓ of 2.9 for the same column with continuous bracing, see Example 8.1-4)

$$P_{nl} := \begin{cases} P_{ne} & \text{if } \lambda_l \le 0.776 \quad \text{(Eq. 1.2.1-5)} \\ \left[1 - 0.15 \cdot \left(\dfrac{P_{crl}}{P_{ne}}\right)^{0.4}\right] \left(\dfrac{P_{crl}}{P_{ne}}\right)^{0.4} \cdot P_{ne} & \text{if } \lambda_l > 0.776 \quad \text{(Eq. 1.2.1-6)} \end{cases}$$

$P_{nl} = 15.2\ kip$ (reduced from 19.4 kips for a column with continuous bracing)

(Continued) 8.1-5 Compressive strength at F_n=37.25 ksi (AISI 2002 Example III-1)

Distortional buckling check per DSM 1.2.1.3

$P_{crd} = 13.1\,\text{kip}$ (from Example 8.1-4)

$$\lambda_d := \sqrt{\frac{P_y}{P_{crd}}} \qquad \lambda_d = 1.92 \qquad \text{(Eq. 1.2.1-10)}$$

$$P_{nd} := \begin{cases} P_y & \text{if } \lambda_d \le 0.561 \\ \left[1 - 0.25\cdot\left(\frac{P_{crd}}{P_y}\right)^{0.6}\right]\left(\frac{P_{crd}}{P_y}\right)^{0.6}\cdot P_y & \text{if } \lambda_d > 0.561 \end{cases} \qquad \begin{matrix}\text{(Eq. 1.2.1-8)} \\ \text{(Eq. 1.2.1-9)}\end{matrix}$$

$P_{nd} = 19.6\,\text{kip}$ (note, the distortional buckling prediction is the same as Example 8.1-2)

Predicted compressive strength per DSM 1.2

$P_n := \min((P_{ne}\ \ P_{nl}\ \ P_{nd}))$ $\qquad P_n = 15.2\,\text{kip}$

The geometry of this section falls within the "pre-qualified" columns of DSM Section 1.1.1.1 and the higher ϕ and lower Ω of DSM Section 1.2.1 may therefore be used.

LRFD: $\phi_c := 0.85$ $\quad \phi_c\cdot P_n = 12.91\,\text{kip}$ compressive design strength

ASD: $\Omega_c := 1.80$ $\quad \frac{P_n}{\Omega_c} = 8.4\,\text{kip}$ compressive allowable strength

8.1-6 Beam-column design (AISI 2002 Example III-1)

AISI (2002) design Example III-1 examines the strength of this 9CS2.25x059 as a beam-column. Consider the same beam-column as calculated via the Direct Strength Method here following the main *Specification* methodology.

Compression: the compressive strength of this section as determined in Example 8.1-5 above.

$\phi_c = 0.85 \qquad P_n = 15.18\,\text{kip}$

for the interaction equation the fully braced compression strength is needed, per Example 8.1-4:

$P_{no} = 19.4\,\text{kip}$

Bending: as discussed in Example 8.1-5 the section is fully braced against lateral and torsional movement, so the flexural strength is that of Example 8.1-1 above

$\phi_b = 0.9 \qquad M_{nx} = 93\,\text{kip}\cdot\text{in}$

Factors to account for approximate 2nd order analysis

$M_{ux} := 55.2\cdot\text{kip}\cdot\text{in}$ First order required strength (moment demand) from AISI 2002 Example III-1

$C_{mx} := 1.0$ The member is pinned at its ends with a load at midspan so the 2nd order (amplified) moments and the primary moments are at the same location and C_m should be 1.0.

$\alpha_x := 1 - \dfrac{P_u}{P_{Ex}}$ α_x is the moment amplification term for strong-axis bending moment. The required strength P_u (demand axial load P_u) is given in Example III-1 as 3.80 kips, the elastic buckling load about the strong axis can be determined by Eq. C5.2.1-6 in the main *Specification*, or taken from finite strip analysis (weak axis and torsion may be restrained to see only strong axis buckling).

$P_u := 3.8\cdot\text{kip}$

$P_{Ex} := 52.1\cdot\text{kip}$ from C5.2.1-6 as used in AISI (2002) Example III-1

or $P_{Ex} := 53.7\cdot\text{kip}$ based on FSM analysis at 240 in., with x movement restricted. This P_{Ex} is used in this solution.

$\alpha_x = 0.93$

Note, the 2nd order required moment (demand) is approximated as: $\dfrac{C_{mx}\cdot M_{ux}}{\alpha_x} = 59.4\,\text{kip}\cdot\text{in}$

Interaction equations

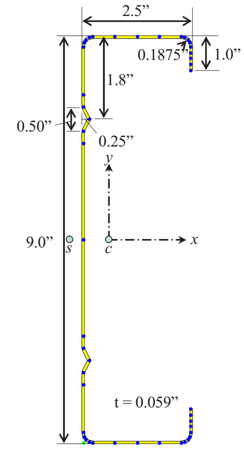
$\dfrac{P_u}{\phi_c\cdot P_n} = 0.29$ which is > 0.15, therefore use Equations C5.2.2-1 and C5.2.2-2

$$\frac{P_u}{\phi_c\cdot P_n} + \frac{C_{mx}\cdot M_{ux}}{\phi_b\cdot M_{nx}\cdot\alpha_x} = 1 \qquad \text{OK, but essentially at maximum value.} \qquad \text{(Eq. C5.2.2-1)}$$

$$\frac{P_u}{\phi_c\cdot P_{no}} + \frac{M_{ux}}{\phi_b\cdot M_{nx}} = 0.89 \qquad \text{OK.} \qquad \text{(Eq. C5.2.2-2)}$$

For format of ASD solution see AISI (2002) Example III-1.

8.2 C-section with lips *modified*



Given:
a. Steel: $F_y = 55$ ksi
b. Section 9CS2.5x059 modified as shown to the right with
lips lengthened to 1 in.
1/4 in. deep stiffeners at 0.2h and 0.8h
c. Finite strip analysis results (Section 3.2.2)
Required:
1. Flexural strength for fully braced member
2. Compressive strength for a fully braced column
3. Compressive strength at F_n=37.25 ksi

8.2-1 Flexural strength for a fully braced member

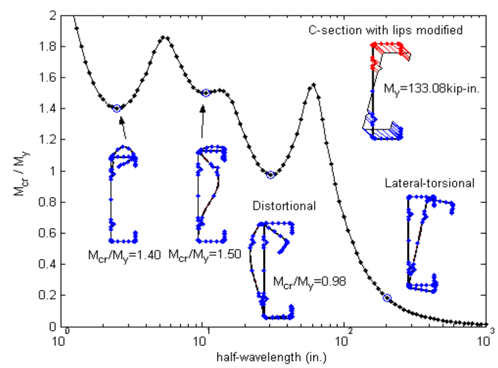
Assume bracing restricts lateral-torsional buckling, but all other modes are free to form.

Inputs from the finite strip analysis include:

$M_y := 133.08 \cdot \text{kip} \cdot \text{in}$

$M_{crl} := 1.40 \cdot M_y \qquad M_{crl} = 186\ \text{kip} \cdot \text{in}$

$M_{crd} := 0.98 \cdot M_y \qquad M_{crd} = 130\ \text{kip} \cdot \text{in}$



Per DSM Section 1.2.2, M_n is the minimum of M_{ne}, $M_{n\ell}$, M_{nd}. For a fully braced member lateral-torsional buckling will not occur and thus $M_{ne} = M_y$, $M_{n\ell}$ and M_{nd} must still be checked.

$M_{ne} := M_y \qquad M_{ne} = 133\ \text{kip} \cdot \text{in} \qquad$ (fully braced)

Local buckling check per DSM 1.2.2.2

$\lambda_l := \sqrt{\dfrac{M_{ne}}{M_{crl}}} \qquad \lambda_l = 0.845 \qquad$ (subscript "l" = "ℓ") (Eq. 1.2.2-7)

$$M_{nl} := \begin{cases} M_{ne} & \text{if } \lambda_l \le 0.776 \\ \left[1 - 0.15 \cdot \left(\dfrac{M_{crl}}{M_{ne}}\right)^{0.4}\right]\left(\dfrac{M_{crl}}{M_{ne}}\right)^{0.4} \cdot M_{ne} & \text{if } \lambda_l > 0.776 \end{cases}$$
(Eq. 1.2.2-5)
(Eq. 1.2.2-6)

$M_{nl} = 126\ \text{kip} \cdot \text{in}$ (this result can be compared with 94 kip-in. for the C-section without the modifications, also note that this member is very nearly "fully effective" in local buckling, that is $M_{n\ell}$ almost equals M_y.)

Distortional buckling check per DSM 1.2.2.3

$\lambda_d := \sqrt{\dfrac{M_y}{M_{crd}}}$ $\quad \lambda_d = 1.01$ (Eq. 1.2.2-10)

$$M_{nd} := \begin{cases} M_y & \text{if } \lambda_d \le 0.673 \\ \left[1 - 0.22\cdot\left(\dfrac{M_{crd}}{M_y}\right)^{0.5}\right]\left(\dfrac{M_{crd}}{M_y}\right)^{0.5}\cdot M_y & \text{if } \lambda_d > 0.673 \end{cases}$$

(Eq. 1.2.2-8)
(Eq. 1.2.2-9)

$M_{nd} = 103\ \text{kip}\cdot\text{in}$ (compare with 93 kip-in. for the unmodified C-section)

Predicted flexural strength per DSM 1.3

$M_n := \min\left(\left(M_{ne}\ \ M_{nl}\ \ M_{nd}\right)\right)$ $\quad M_n = 103\ \text{kip}\cdot\text{in}$

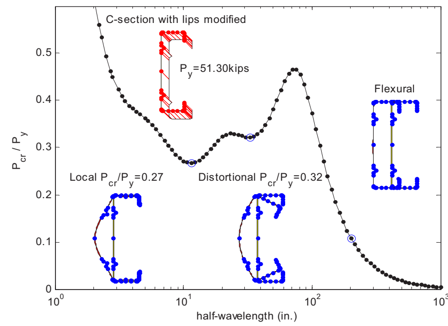
The geometry of this section does not fall within the "pre-qualified" beams of DSM Section 1.1.1.2 and thus the rational analysis values for ϕ and Ω of main *Spec*. section A1.1(b) apply.

LRFD: $\phi_b := 0.8$ $\quad \phi_b\cdot M_n = 82\ \text{kip}\cdot\text{in}$ flexural design strength

ASD: $\Omega_b := 2.00$ $\quad \dfrac{M_n}{\Omega_b} = 52\ \text{kip}\cdot\text{in}$ flexural allowable strength

(The use of the more conservative rational analysis values for ϕ and Ω result in the design strength ϕM_n of the modified section being essentially the same as the unmodified section. The primary benefit in bending was derived from the lengthening of the lip, and this simple change (without the web stiffeners) would have still allowed the member to be pre-qualified for ϕ. An alternative may be to use a single web stiffener, instead of two, that has been pre-qualified in Section 1.1.1.2 of DSM, this may provide a more economical alternative since a higher ϕ factor may be used. Finally, product development and further discussion of pre-qualified members is presented in Chapter 7 of this Design Guide. It may be worthwhile to pursue work to add a beneficial member to the list of pre-qualified sections.)

8.2-2 Compressive strength for a continuously braced column



Inputs from the finite strip analysis include:

$P_y := 51.30 \cdot kip$

$P_{crl} := 0.27 \cdot P_y$ $\quad$ $P_{crl} = 13.9\,kip$

$P_{crd} := 0.32 \cdot P_y$ $\quad$ $P_{crd} = 16.4\,kip$

Per DSM 1.2.1, P_n is the minimum of P_{ne}, $P_{n\ell}$, P_{nd}. If a column is continuously braced then global buckling P_{ne} is restricted and the squash load will develop:

$P_{ne} := P_y$ $\quad$ $P_{ne} = 51.3\,kip$

Local buckling check per DSM 1.2.1.2

$$\lambda_l := \sqrt{\frac{P_{ne}}{P_{crl}}} \qquad \lambda_l = 1.925 \qquad \text{(subscript "l" = "}\ell\text{")} \qquad \text{(Eq. 1.2.1-7)}$$

$$P_{nl} := \begin{cases} P_{ne} & \text{if } \lambda_l \le 0.776 \\ \left[1 - 0.15\cdot\left(\frac{P_{crl}}{P_{ne}}\right)^{0.4}\right]\left(\frac{P_{crl}}{P_{ne}}\right)^{0.4}\cdot P_{ne} & \text{if } \lambda_l > 0.776 \end{cases} \qquad \text{(Eq. 1.2.1-5)} \; \text{(Eq. 1.2.1-6)}$$

$P_{nl} = 27.7\,kip$ (compare with 19.4 kips for the unmodified C-section)

Distortional buckling check per DSM 1.2.1.3

$$\lambda_d := \sqrt{\frac{P_y}{P_{crd}}} \qquad \lambda_d = 1.768 \qquad \text{(Eq. 1.2.1-10)}$$

$$P_{nd} := \begin{cases} P_y & \text{if } \lambda_d \le 0.561 \\ \left[1 - 0.25\cdot\left(\frac{P_{crd}}{P_y}\right)^{0.6}\right]\left(\frac{P_{crd}}{P_y}\right)^{0.6}\cdot P_y & \text{if } \lambda_d > 0.561 \end{cases} \qquad \text{(Eq. 1.2.1-8)} \; \text{(Eq. 1.2.1-9)}$$

$P_{nd} = 22.6\,kip$ (compare with 19.6 kips for the unmodified C-section)

Predicted compressive strength per DSM 1.2

$P_n := \min\left(\left(P_{ne} \;\; P_{nl} \;\; P_{nd}\right)\right)$ $\quad$ $P_n = 22.6\,kip$

The geometry of this section falls outside the "pre-qualified" columns of DSM Section 1.1.1.1 and thus the rational analysis values for ϕ and Ω of main *Specification* section A1.1(b) apply.

LRFD: $\phi_c := 0.80$ $\quad$ $\phi_c \cdot P_n = 18.1\,kip$

ASD: $\Omega_c := 2.00$ $\quad$ $\frac{P_n}{\Omega_c} = 11.3\,kip$

(Even with the more conservative ϕ values from main *Specification* A1.1.(b), as opposed to the pre-qualified values in DSM 1.1.1.1, the modified C-section has a higher design strength than the original C-section.)

8.2-3 Compressive strength at F_n=37.25 ksi

In Example 8.1-5 the compressive strength of an unmodified C-section at a uniform compressive stress of 37.25 ksi was considered. This stress was determined based on strong axis buckling at an unbraced length of 20 ft. as detailed in AISI (2002) Example III-1.

Similar to Example 8.1-5 the strength of the column is calculated at a long column stress F_n of 37.25 ksi. Per DSM Section 1.2.1, P_n is the minimum of P_{ne}, $P_{n\ell}$, P_{nd}, and:

$P_{ne} := A_g \cdot 37.25 \cdot ksi$ $\qquad P_{ne} = 34.7\,kip$

Note, P_{ne} is slightly larger in this example than in Example 8.1-5 because the cross-sectional area of the *modified* C-section is larger.

Local buckling check per DSM 1.2.1.2

$$\lambda_l := \sqrt{\frac{P_{ne}}{P_{crl}}} \qquad \lambda_l = 1.584 \qquad \text{(subscript "l" = "}\ell\text{")} \qquad \text{(Eq. 1.2.1-7)}$$

$$P_{nl} := \begin{cases} P_{ne} & \text{if } \lambda_l \le 0.776 \\ \left[1 - 0.15\cdot\left(\frac{P_{crl}}{P_{ne}}\right)^{0.4}\right]\left(\frac{P_{crl}}{P_{ne}}\right)^{0.4}\cdot P_{ne} & \text{if } \lambda_l > 0.776 \end{cases} \qquad \begin{matrix}\text{(Eq. 1.2.1-5)}\\ \text{(Eq. 1.2.1-6)}\end{matrix}$$

$P_{nl} = 21.6\,kip$ (down from 27.7 kips when P_{ne}=P_y, but well above $P_{n\ell}$ = 15.2 kips for the unmodified C-section.)

Distortional buckling check per DSM 1.2.1.3

$P_{nd} = 22.6\,kip$ identical to that calculated in Example 8.2-2

Predicted compressive strength per DSM 1.2

$P_n := \min((P_{ne}\ \ P_{nl}\ \ P_{nd}))$ $\qquad P_n = 21.6\,kip$

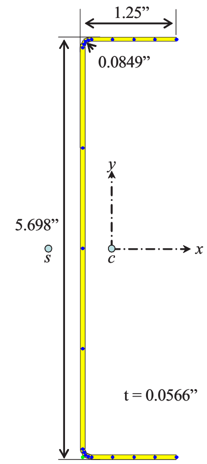
The geometry of this section falls outside the "pre-qualified" columns of DSM Section 1.1.1.1 and thus the rational analysis values for ϕ and Ω of main *Specification* section A1.1(b) apply.

LRFD: $\phi_c := 0.80 \quad \phi_c \cdot P_n = 17.2\,kip$

ASD: $\Omega_c := 2.00 \quad \frac{P_n}{\Omega_c} = 10.8\,kip$

(Note, the predicted design strength is well above the unmodified C-section, predicted to have a nominal strength of only 12.9 kips.)

8.3 C-section without lips (track section)



Given:

a. Steel: $F_y = 33$ ksi

b. Section SSMA Track 550T125-54 as shown to the right

c. Finite strip analysis results (Section 3.2.3)

Required:

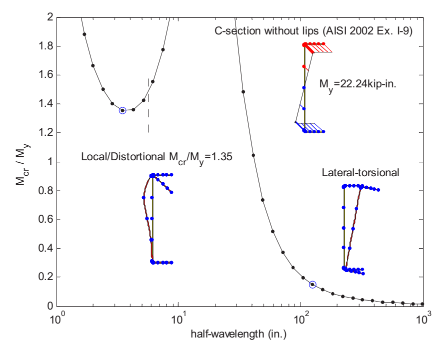
1. Flexural strength about strong axis for a fully braced member (AISI 2002 Example I-9)
2. Flexural strength at F_c=30.93 ksi (AISI 2002 Example II-3)
3. Compressive strength for a continuously braced column
4. Compressive strength for L=49.3 in.
5. Flexural strength about weak-axis for L=49.3 in.
6. Beam-column design strength check for L=49.3 in.

8.3-1 Flexural strength about strong axis for a fully braced member (AISI 2002 Example I-9)

Determination of the flexural strength for a fully braced member is equivalent to determining the effective section modulus at yield in the main *Specification*. See AISI (2002) Example I-9.

Finite strip analysis of 550T125-54 in pure bending as summarized in Section 3.2.3, and below.

Inputs from the finite strip analysis include:



$M_y := 22.24 \cdot \text{kip} \cdot \text{in}$

$M_{crl} := 1.35 \cdot M_y \qquad M_{crl} = 30\,\text{kip} \cdot \text{in}$

$M_{crd} := 1.35 \cdot M_y \qquad M_{crd} = 30\,\text{kip} \cdot \text{in}$

Here the conservative assumption is made that the first minimum observed could be either local or distortional buckling. Based on the commentary to DSM (Appendix 1) the wavelength could be used to identify this as a local mode, but the similarity with distortional buckling of a lipped channel is obvious, so it was decided here to conservatively assume the observed mode could be either local or distortional.

For a fully braced member LTB will not occur and thus $M_{ne} = M_y$, $M_{n\ell}$ and M_{nd} must still be checked.

$M_{ne} := M_y \qquad M_{ne} = 22\,\text{kip} \cdot \text{in} \qquad$ (fully braced)

Local buckling check per DSM 1.2.2.2

$$\lambda_l := \sqrt{\frac{M_{ne}}{M_{crl}}} \qquad \lambda_l = 0.86 \qquad (\text{subscript "l" = "}\ell\text{"}) \qquad \text{(Eq. 1.2.2-7)}$$

$$M_{nl} := \begin{cases} M_{ne} & \text{if } \lambda_l \le 0.776 \\ \left[1 - 0.15 \cdot \left(\frac{M_{crl}}{M_{ne}}\right)^{0.4}\right]\left(\frac{M_{crl}}{M_{ne}}\right)^{0.4} \cdot M_{ne} & \text{if } \lambda_l > 0.776 \end{cases} \qquad \text{(Eq. 1.2.2-5)} \quad \text{(Eq. 1.2.2-6)}$$

$M_{nl} = 21\,\text{kip} \cdot \text{in}$

Distortional buckling check per DSM 1.2.2.3

$\lambda_d := \sqrt{\frac{M_y}{M_{crd}}}$ $\lambda_d = 0.86$ (Eq. 1.2.2-10)

$$M_{nd} := \begin{cases} M_y & \text{if } \lambda_d \le 0.673 \\ \left[1 - 0.22 \cdot \left(\frac{M_{crd}}{M_y}\right)^{0.5}\right]\left(\frac{M_{crd}}{M_y}\right)^{0.5} \cdot M_y & \text{if } \lambda_d > 0.673 \end{cases}$$

(Eq. 1.2.2-8)
(Eq. 1.2.2-9)

$M_{nd} = 19\,\text{kip}\cdot\text{in}$ In this example the distortional buckling strength is only 2 kip-in. less than the local buckling strength, so the conservative assumption that the first minimum might be considered either local or distortional is not overly detrimental to the economy of the design. If the (assumed) lateral bracing that is restricting LTB also restricts the flange movement, it would be reasonable to assume DB is restricted here as well, and thus M_{nd}=M_y and $M_{n\ell}$ would control. Further, since only local buckling interacts with LTB for longer lengths $M_{n\ell}$ will control over M_{nd} regardless. Thus, the conservative assumption for M_{crd} only has impact for intermediate lengths spans for which LTB is braced.

Predicted flexural strength per 1.3

$M_n := \min((M_{ne}\ \ M_{nl}\ \ M_{nd}))$ $M_n = 19\,\text{kip}\cdot\text{in}$

The geometry of this section falls outside the "pre-qualified" beams of DSM 1.1.1.2 and thus the ϕ and Ω of the rational analysis clause of the main *Specification*, A1.1(b), must be used.

LRFD: $\phi_b := 0.8$ $\phi_b \cdot M_n = 15\,\text{kip}\cdot\text{in}$ flexural design strength

ASD: $\Omega_b := 2.00$ $\frac{M_n}{\Omega_b} = 10\,\text{kip}\cdot\text{in}$ flexural allowable strength

8.3-2 Flexural strength for F_c=30.93 ksi (AISI 2002 Example II-3)

In AISI (2002) Example II-3 this track section is examined as a beam with a 72 in. simple span under uniform load, braced against twisting and lateral deflection at the ends and at midspan. Following main *Specification* Equations C3.1.2.1 AISI (2002) Example II-3 provides the elastic lateral-torsional buckling stress F_e and then the inelastic lateral-torsional buckling stress F_c.

Method (a) per AISI (2002) Example II-3

$C_b := 1.30$ moment gradient modification factor for a half-span under uniform load (C3.1.2.1-10)

$F_e := C_b \cdot 36.48 \cdot ksi$ $F_e = 47.42\ ksi$ determined via C3.1.2.1-5

$F_c := 30.93 \cdot ksi$ determined via C3.1.2.1-3

Method (b) from finite strip analysis

$M_{cre} := 1.32 \cdot M_y$ direct from finite strip analysis at a half-wavelength of 36 in., see Section 3.2.3

$M_{cre} := C_b \cdot M_{cre}$ C_b of the main *Spec*. applies, and $M_{cre} = 38.16\ kip \cdot in$

to compare with F_e, divide by S_g $\frac{M_{cre}}{S_g} = 56.63\ ksi$

$$M_{ne} := \begin{cases} M_{cre} & \text{if } M_{cre} < 0.56 \cdot M_y \quad \text{(Eq. 1.2.2-1)} \\ \frac{10}{9} \cdot M_y \cdot \left(1 - \frac{10 \cdot M_y}{36 \cdot M_{cre}}\right) & \text{if } 2.78 \cdot M_y \ge M_{cre} \ge 0.56 \cdot M_y \quad \text{(Eq. 1.2.2-2)} \\ M_y & \text{if } M_{cre} > 2.78 \cdot M_y \quad \text{(Eq. 1.2.2-3)} \end{cases}$$

$M_{ne} = 20.71\ kip \cdot in$ to compare with F_c divide by S_g $\frac{M_{ne}}{S_g} = 30.73\ ksi$

To be consistent with the AISI (2002) *Design Manual* example base the lateral-torsional buckling on strength Method (a) the stress F_c: $M_{ne} = S_g F_c$. Note, $M_{n\ell}$ and M_{nd} must still be checked.

$S_g = 0.67\ in^3$ $M_{ne} := S_g \cdot 30.93 \cdot ksi$ $M_{ne} = 21\ kip \cdot in$ (fully braced)

Note: For a long unbraced length lateral-torsional buckling (LTB) controls the strength of beams. In the main *Specification* interaction of local and lateral-torsional buckling is handled by calculating the maximum stress for LTB (e.g., F_c, Eq. C3.1.2.1-3) and then determining the effective section modulus, S_c, at stress F_c. The maximum stress for LTB can be converted into a strength by multiplying the stress F_c times the gross section modulus, i.e., M_{ne}=$S_g F_c$. In DSM the local buckling equation then picks up this interaction by modifying the slenderness (DSM Eq.1.2.2-7) and the maximum strength (DSM Eqs. 1.2.2-5,6).

(Continued) 8.3-2 Flexural strength at F_c=30.93 ksi

Local buckling check per DSM 1.2.2.2

$$\lambda_l := \sqrt{\frac{M_{ne}}{M_{crl}}} \qquad \lambda_l = 0.83 \qquad \text{(subscript "l" = "}\ell\text{")} \qquad \text{(Eq. 1.2.2-7)}$$

$$M_{nl} := \begin{cases} M_{ne} & \text{if } \lambda_l \le 0.776 \\ \left[1 - 0.15\cdot\left(\frac{M_{crl}}{M_{ne}}\right)^{0.4}\right]\left(\frac{M_{crl}}{M_{ne}}\right)^{0.4}\cdot M_{ne} & \text{if } \lambda_l > 0.776 \end{cases} \qquad \begin{matrix}\text{(Eq. 1.2.2-5)}\\ \text{(Eq. 1.2.2-6)}\end{matrix}$$

$M_{nl} = 20\,kip\cdot in$ the local-LTB reduction is quite small in this case, M_{ne}= 21kip-in.

Distortional buckling check per DSM 1.2.2.3

$$\lambda_d := \sqrt{\frac{M_y}{M_{crd}}} \qquad \lambda_d = 0.86 \qquad \text{(Eq. 1.2.2-10)}$$

$$M_{nd} := \begin{cases} M_y & \text{if } \lambda_d \le 0.673 \\ \left[1 - 0.22\cdot\left(\frac{M_{crd}}{M_y}\right)^{0.5}\right]\left(\frac{M_{crd}}{M_y}\right)^{0.5}\cdot M_y & \text{if } \lambda_d > 0.673 \end{cases} \qquad \begin{matrix}\text{(Eq. 1.2.2-8)}\\ \text{(Eq. 1.2.2-9)}\end{matrix}$$

$M_{nd} = 19\,kip\cdot in$ Note, the assumption that the track section may undergo distortional buckling is still governing the strength (as in Example 8.3-1 above), but M_{nd} is independent of M_{ne} - so if F_c is even slightly lower $M_{n\ell}$ will soon govern the strength prediction. Therefore the "conservatism" of including the first minimum as either local or distortional only governs over a short length. For more on this type of behavior see Chapter 4 and Example 8.13 of this Guide.

Predicted flexural strength per DSM 1.3

$$M_n := \min((M_{ne}\ \ M_{nl}\ \ M_{nd})) \qquad M_n = 19\,kip\cdot in$$

The geometry of this section falls outside the "pre-qualified" beams of DSM 1.1.1.2 and thus the ϕ and Ω of the rational analysis clause of the main *Specification*, A1.1(b), must be used.

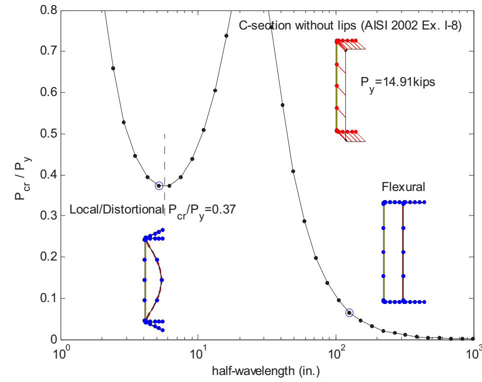
LRFD: $\phi_b := 0.8 \qquad \phi_b\cdot M_n = 15\,kip\cdot in$

ASD: $\Omega_b := 2.00 \qquad \frac{M_n}{\Omega_b} = 10\,kip\cdot in$

Further Notes on AISI (2002) Example II-3: The design check for shear could follow the nomenclature of Section 4.4.1 of this Guide, or that of AISI (2002) Example II-3, and arrive at the same final result.

8.3-3 Compressive strength for a continuously braced column

Finite strip analysis of 550T125-54 in pure compression:



Inputs from the finite strip analysis include:

$P_y := 14.91 \cdot kip$

$P_{crl} := 0.37 \cdot P_y$ $\qquad P_{crl} = 5.5\,kip$

$P_{crd} := 0.37 \cdot P_y$ $\qquad P_{crd} = 5.5\,kip$

Per DSM 1.2.1, P_n is the minimum of P_{ne}, $P_{n\ell}$, P_{nd}. If a column is continuously braced then global buckling is restricted and the squash load will develop if the section is compact.

$P_{ne} := P_y$ $\qquad P_{ne} = 14.9\,kip$

Local buckling check per DSM 1.2.1.2

$$\lambda_l := \sqrt{\frac{P_{ne}}{P_{crl}}} \qquad \lambda_l = 1.64 \qquad \text{(subscript "l" = "}\ell\text{")} \qquad \text{(Eq. 1.2.1-7)}$$

$$P_{nl} := \begin{cases} P_{ne} & \text{if } \lambda_l \le 0.776 \\ \left[1 - 0.15\cdot\left(\frac{P_{crl}}{P_{ne}}\right)^{0.4}\right]\left(\frac{P_{crl}}{P_{ne}}\right)^{0.4}\cdot P_{ne} & \text{if } \lambda_l > 0.776 \end{cases} \qquad \begin{matrix}\text{(Eq. 1.2.1-5)}\\ \text{(Eq. 1.2.1-6)}\end{matrix}$$

$P_{nl} = 9.0\,kip$

Distortional buckling check per DSM 1.2.1.3

$$\lambda_d := \sqrt{\frac{P_y}{P_{crd}}} \qquad \lambda_d = 1.64 \qquad \text{(Eq. 1.2.1-10)}$$

$$P_{nd} := \begin{cases} P_y & \text{if } \lambda_d \le 0.561 \\ \left[1 - 0.25\cdot\left(\frac{P_{crd}}{P_y}\right)^{0.6}\right]\left(\frac{P_{crd}}{P_y}\right)^{0.6}\cdot P_y & \text{if } \lambda_d > 0.561 \end{cases} \qquad \begin{matrix}\text{(Eq. 1.2.1-8)}\\ \text{(Eq. 1.2.1-9)}\end{matrix}$$

$P_{nd} = 7.1\,kip$ Inclusion of this distortional buckling (DB) check presumes (1) the first minimum identified could be DB or local, (2) the bracing that is restricting long column buckling is not restricting DB.

Predicted compressive strength per DSM 1.2

$P_n := \min\left(\left(P_{ne}\ \ P_{nl}\ \ P_{nd}\right)\right)$ $\qquad P_n = 7.1\,kip$

The geometry of this section falls outside the "pre-qualified" columns of DSM 1.1.1.1 and thus the ϕ and Ω of the rational analysis clause of the main *Specification*, A1.1(b), must be used.

LRFD: $\phi_c := 0.80 \qquad \phi_c \cdot P_n = 5.7\,kip$

ASD: $\Omega_c := 2.00 \qquad \frac{P_n}{\Omega_c} = 3.5\,kip$

8.3-4 Compressive strength at L=49.3 in.

Assume concentric loading, simply-supported ends, bracing $KL_x = KL_y = KL_t = 49.3$ in.

Flexural, torsional, or torsional-flexural check per DSM 1.2.1.1

The weak-axis flexural buckling is read directly from the FSM analysis at L=49.3 in.

$P_{cre} := 0.408 \cdot P_y \qquad P_{cre} = 6.08\,\text{kip}$

$$\lambda_c := \sqrt{\frac{P_y}{P_{cre}}} \qquad \text{(Eq. 1.2.1-3)}$$

$$P_{ne} := \begin{cases} 0.658^{\lambda_c^2} \cdot P_y & \text{if } \lambda_c \le 1.5 \quad \text{(Eq. 1.2.1-1)} \\ \dfrac{.877}{\lambda_c^2} \cdot P_y & \text{if } \lambda_c > 1.5 \quad \text{(Eq. 1.2.1-2)} \end{cases} \qquad P_{ne} = 5.34\,\text{kip}$$

Local buckling check per DSM 1.2.1.2

$$\lambda_l := \sqrt{\frac{P_{ne}}{P_{crl}}} \qquad \lambda_l = 0.98 \qquad \text{(subscript "l" = "}\ell\text{")} \qquad \text{(Eq. 1.2.1-7)}$$

$$P_{nl} := \begin{cases} P_{ne} & \text{if } \lambda_l \le 0.776 \quad \text{(Eq. 1.2.1-5)} \\ \left[1 - 0.15 \cdot \left(\dfrac{P_{crl}}{P_{ne}}\right)^{0.4}\right]\left(\dfrac{P_{crl}}{P_{ne}}\right)^{0.4} \cdot P_{ne} & \text{if } \lambda_l > 0.776 \quad \text{(Eq. 1.2.1-6)} \end{cases} \qquad P_{nl} = 4.6\,\text{kip}$$

Distortional buckling check per DSM 1.2.1.3

$$\lambda_d := \sqrt{\frac{P_y}{P_{crd}}} \qquad \lambda_d = 1.64 \qquad \text{(Eq. 1.2.1-10)}$$

$$P_{nd} := \begin{cases} P_y & \text{if } \lambda_d \le 0.561 \quad \text{(Eq. 1.2.1-8)} \\ \left[1 - 0.25 \cdot \left(\dfrac{P_{crd}}{P_y}\right)^{0.6}\right]\left(\dfrac{P_{crd}}{P_y}\right)^{0.6} \cdot P_y & \text{if } \lambda_d > 0.561 \quad \text{(Eq. 1.2.1-9)} \end{cases}$$

$P_{nd} = 7.1\,\text{kip}$ Inclusion of this distortional buckling (DB) check presumes (1) the first minimum identified could be DB or local, (2) the bracing that is restricting long column buckling is not restricting DB. DB does not control.

Predicted compressive strength per DSM 1.2

$P_n := \min((P_{ne}\ \ P_{nl}\ \ P_{nd})) \qquad P_n = 4.6\,\text{kip}$

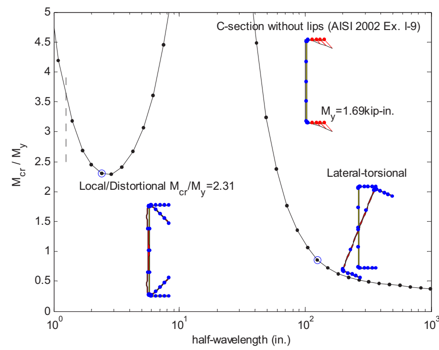
The geometry of this section falls outside the "pre-qualified" columns of DSM 1.1.1.1 and thus the ϕ and Ω of the rational analysis clause of the main Specification, A1.1(b), must be used.

LRFD: $\phi_c := 0.80 \qquad \phi_c \cdot P_n = 3.7\,\text{kip}$

ASD: $\Omega_c := 2.00 \qquad \dfrac{P_n}{\Omega_c} = 2.3\,\text{kip}$

8.3-5 Flexural strength about weak-axis (flange tips in compression) for L=49.3 in.

<u>Flange tips in compression for 550T125-54</u>



Inputs from the finite strip analysis:

$M_y := 1.69 \cdot kip \cdot in$

$M_{crl} := 2.31 \cdot M_y$ $\qquad M_{crl} = 3.9\, kip \cdot in$

$M_{crd} := 2.31 \cdot M_y$ $\qquad M_{crd} = 3.9\, kip \cdot in$

$M_{cre} := 3.24 \cdot M_y$ $\qquad M_{cre} = 5.48\, kip \cdot in$

From Section 2.2 of this Guide, $M_{cr\ell}$>1.66M_y and M_{crd}>2.21M_y and M_{cre}>2.78M_y therefore no reductions will occur for local, distortional, or global buckling and the design strength will equal the yield moment for these sections.

More formally, the equations of Appendix 1 may be used to come to the same conclusion.

$M_{ne} := M_y$ $\qquad M_{ne} = 1.69\, kip \cdot in$ $\qquad$ (for laterally braced member)

$\lambda_l \le 0.776$ so $\quad M_{nl} := M_{ne}$ $\qquad M_{nl} = 1.69\, kip \cdot in$ $\qquad$ (Eq. 1.2.2-5)

$\lambda_d \le 0.673$ so $\quad M_{nd} := M_y$ $\qquad M_{nd} = 1.69\, kip \cdot in$ $\qquad$ (Eq. 1.2.2-8)

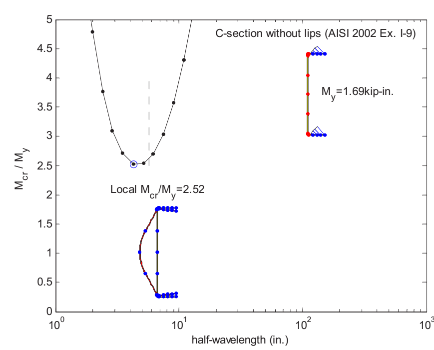
Predicted compressive strength per DSM 1.3

$M_n := \min((M_{ne} \;\; M_{nl} \;\; M_{nd}))$ $\qquad M_n = 1.69\, kip \cdot in$

The geometry of this section falls outside the "pre-qualified" beams of 1.1.1.2 and thus the ϕ and Ω of the rational analysis clause of the main *Specification*, A1.1(b), must be used.

LRFD: $\quad \phi_b := 0.8 \quad \phi_b \cdot M_n = 1.35\, kip \cdot in$ $\qquad$ ASD: $\quad \Omega_b := 2.00 \quad \dfrac{M_n}{\Omega_b} = 0.85\, kip \cdot in$

<u>Flange tips in tension for 550T125-54</u>



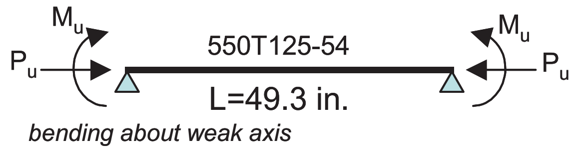
Inputs from the finite strip analysis:

$M_y := 1.69 \cdot kip \cdot in$

$M_{crl} := 2.52 \cdot M_y$ $\qquad M_{crl} = 4.26\, kip \cdot in$

Distortional buckling does not occur in this section. From Section 2.2 of this guide, $M_{cr\ell}$>1.66M_y and M_{cre}>2.78M_y therefore no reductions will occur for local or global buckling and the design strength will equal the yield moment for this section.

8.3-6 Beam-column design strength for L=49.3 in., and P_u = 2.8 kip and M_u = 0.32 kip-in.



$P_u = 2.8\,\text{kip}$

$M_u = 0.32\,\text{kip}\cdot\text{in}$ (1st order moment)

Compression: the compressive strength of this section as determined in Example 8.3-4 above.

$\phi_c = 0.8$ $P_n = 4.58\,\text{kip}$

for the interaction equation the fully braced compressive strength is needed, per Example 8.3-3

$P_{no} = 7.1\,\text{kip}$

Bending: the flexural strength is that of Example 8.3-5 above

$\phi_b = 0.8$ $M_n = 1.69\,\text{kip}\cdot\text{in}$

Factors to account for approximate 2nd order analysis

$M_u = 0.32\,\text{kip}\cdot\text{in}$ 1st order moment demand

$C_m := 1.0$ The member is pinned at its ends with a uniform moment so the 2nd order (amplified) moments and the primary moments are at the same location and C_m should be 1.0.

$\alpha := 1 - \frac{P_u}{P_E}$ α is the moment amplification term for weak-axis bending. The demand axial load P_u is 2.8 kips, the elastic buckling load about the weak axis can be determined by formula, or taken from a finite strip analysis.

$P_u = 2.8\,\text{kip}$ $P_E := P_{cre}$ $P_E = 6.08\,\text{kip}$ based on FSM analysis at 49.3 in. per Example 8.3-4

$\alpha = 0.54$

note, the 2nd order required moment demand is approximated as $\frac{C_m \cdot M_u}{\alpha} = 0.59\,\text{kip}\cdot\text{in}$ this is the flexural required strength

Interaction equations

$\frac{P_u}{\phi_c \cdot P_n} = 0.76$ which is > 0.15, therefore use equations C5.2.2-1 and C5.2.2-2

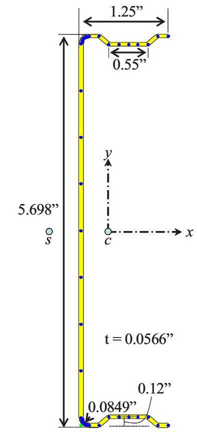
$$\frac{P_u}{\phi_c \cdot P_n} + \frac{C_m \cdot M_u}{\phi_b \cdot M_n \cdot \alpha} = 1.2 \quad \textbf{NG!} \qquad \text{(Eq. C5.2.2-1)}$$

$$\frac{P_u}{\phi_c \cdot P_{no}} + \frac{M_u}{\phi_b \cdot M_n} = 0.73 \quad \text{OK.} \qquad \text{(Eq. C5.2.2-2)}$$

Based on a conventional beam-column interaction check, but using the design strengths determined by the Direct Strength Method, this cross-section fails under the applied load. A stronger cross-section is needed for this loading.

A direct beam-column analysis, under the actual applied stresses, using a DSM procedure that is anticipated for use in the future, is provided in Section 6.3.1 of this Guide.

8.4 C-section without lips *modified* (track section)



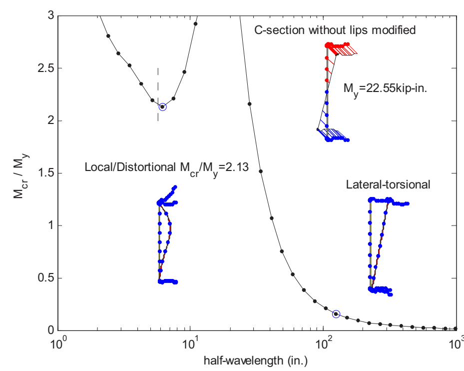
Given:
a. Steel: $F_y = 33$ ksi
b. *Modified* Section SSMA Track 550T125-54
- stiffener added to the flanges as shown

c. Finite strip analysis results (Section 3.2.4)
Required:
1. Flexural strength about strong axis for a fully braced member
2. Compressive strength for a continuously braced column

8.4-1 Flexural strength about strong axis for a fully braced member

Finite strip analysis of modified 550T125-54 in pure bending is summarized in Section 3.2.4 and below.

Inputs from the finite strip analysis include:



$M_y := 22.55 \cdot \text{kip} \cdot \text{in}$

$M_{crl} := 2.13 \cdot M_y$ $\qquad M_{crl} = 48\,\text{kip} \cdot \text{in}$

$M_{crd} := 2.13 \cdot M_y$ $\qquad M_{crd} = 48\,\text{kip} \cdot \text{in}$

Similar to Example 8.3 the conservative assumption is again made that the first minimum observed could be either local or distortional buckling.

For a fully braced member LTB will not occur and thus $M_{ne} = M_y$, $M_{n\ell}$ and M_{nd} must still be checked.

$M_{ne} := M_y$ $\qquad M_{ne} = 23\,\text{kip} \cdot \text{in}$ $\qquad$ (fully braced)

Local buckling check per DSM 1.2.2.2

Per Sec. 2.2 of this Guide, since $M_{cr\ell} > 1.66 M_y$ $\quad M_{nl} := M_{ne}$ $\quad M_{nl} = 23\,\text{kip} \cdot \text{in}$ $\qquad$ (Eq. 1.2.2-5)

Distortional buckling check per DSM 1.2.2.3

$$\lambda_d := \sqrt{\frac{M_y}{M_{crd}}} \qquad \lambda_d = 0.69 \qquad \text{(Eq. 1.2.2-10)}$$

$$M_{nd} := \begin{cases} M_y & \text{if } \lambda_d \le 0.673 \\ \left[1 - 0.22 \cdot \left(\frac{M_{crd}}{M_y}\right)^{0.5}\right]\left(\frac{M_{crd}}{M_y}\right)^{0.5} \cdot M_y & \text{if } \lambda_d > 0.673 \end{cases} \qquad \begin{array}{l}\text{(Eq. 1.2.2-8)} \\ \text{(Eq. 1.2.2-9)}\end{array}$$

$M_{nd} = 22\,\text{kip} \cdot \text{in}$ $\quad$ The DB reduction from M_y is small. Strength is improved from Example 8.3-1.

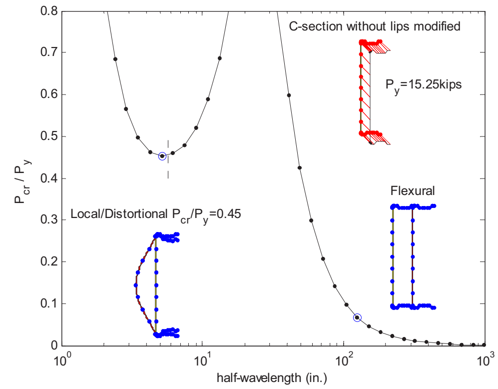
Predicted flexural strength per DSM 1.3 $\quad M_n := \min((M_{ne}\ \ M_{nl}\ \ M_{nd}))$ $\quad M_n = 22\,\text{kip} \cdot \text{in}$

The geometry of this section falls outside the "pre-qualified" beams of DSM 1.1.1.2 and thus the ϕ and Ω of the rational analysis clause of the main *Specification,* A1.1(b), must be used.

LRFD: $\quad \phi_b := 0.8$ $\quad \phi_b \cdot M_n = 18\,\text{kip} \cdot \text{in}$ $\qquad$ ASD: $\quad \Omega_b := 2.00$ $\quad \frac{M_n}{\Omega_b} = 11.17\,\text{kip} \cdot \text{in}$

8.4-2 Compressive strength for a continuously braced column

Finite strip analysis of modified 550T125-54 in compression as summarized in Section 3.2.4.



Inputs from the finite strip analysis include:

$P_y := 15.25 \cdot kip$

$P_{crl} := 0.45 \cdot P_y$ $\qquad P_{crl} = 6.9\,kip$

$P_{crd} := 0.45 \cdot P_y$ $\qquad P_{crd} = 6.9\,kip$

Per DSM 1.2.1, P_n is the minimum of P_{ne}, $P_{n\ell}$, P_{nd}. Assume the column is continuously braced against global buckling, then the global column strength P_{ne} is the squash load.

$P_{ne} := P_y$ $\qquad P_{ne} = 15.2\,kip$

Local buckling check per DSM 1.2.1.2

$\lambda_l := \sqrt{\dfrac{P_{ne}}{P_{crl}}}$ $\qquad \lambda_l = 1.49$ $\qquad$ (subscript "l" = "ℓ") $\qquad$ (Eq. 1.2.1-7)

$$P_{nl} := \begin{cases} P_{ne} & \text{if } \lambda_l \le 0.776 \\ \left[1 - 0.15 \cdot \left(\dfrac{P_{crl}}{P_{ne}}\right)^{0.4}\right]\left(\dfrac{P_{crl}}{P_{ne}}\right)^{0.4} \cdot P_{ne} & \text{if } \lambda_l > 0.776 \end{cases}$$

(Eq. 1.2.1-5)
(Eq. 1.2.1-6)

$P_{nl} = 9.9\,kip$

Distortional buckling check per DSM 1.2.1.3

$\lambda_d := \sqrt{\dfrac{P_y}{P_{crd}}}$ $\qquad \lambda_d = 1.49$ $\qquad$ (Eq. 1.2.1-10)

$$P_{nd} := \begin{cases} P_y & \text{if } \lambda_d \le 0.561 \\ \left[1 - 0.25 \cdot \left(\dfrac{P_{crd}}{P_y}\right)^{0.6}\right]\left(\dfrac{P_{crd}}{P_y}\right)^{0.6} \cdot P_y & \text{if } \lambda_d > 0.561 \end{cases}$$

(Eq. 1.2.1-8)
(Eq. 1.2.1-9)

$P_{nd} = 8.0\,kip$ Inclusion of this distortional buckling (DB) check presumes (1) the first minimum identified could be DB or local, (2) the bracing which is restricting long column buckling is not restricting DB.

Predicted compressive strength per DSM 1.2

$P_n := \min\left(\left(P_{ne} \;\; P_{nl} \;\; P_{nd}\right)\right)$ $\qquad P_n = 7.98\,kip$

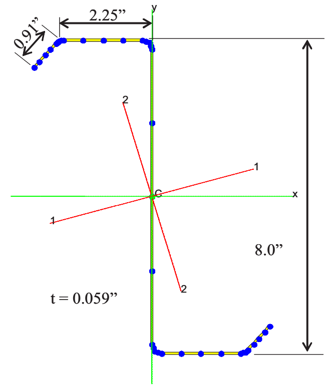
The geometry of this section falls outside the "pre-qualified" columns of DSM 1.1.1.1 and thus the ϕ and Ω of the rational analysis clause, main *Specification* A1.1(b) must be used.

LRFD: $\qquad \phi_c := 0.80 \quad \phi_c \cdot P_n = 6.4\,kip$ $\qquad$ compressive design strength

ASD: $\qquad \Omega_c := 2.00 \quad \dfrac{P_n}{\Omega_c} = 4\,kip$ $\qquad$ compressive allowable strength

8.5 Z-section with lips

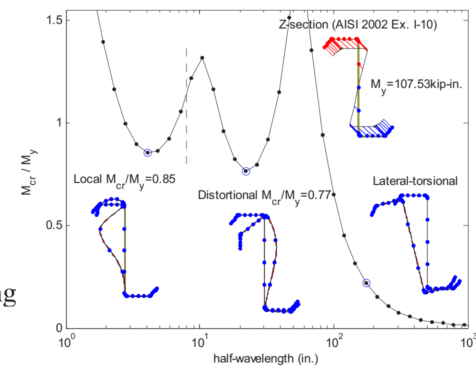
Given:
a. Steel: F_y = 55 ksi
b. Section 8ZS2.25x059 as shown to the right
c. Finite strip analysis results (Section 3.2.5)
Required:
1. Flexural strength for a fully braced member
2. Flexural strength for L=48.5 in. (AISI 2002 Ex. II-2)
3. Compressive strength for a continuously braced column
4. Compressive strength at F_n=25.9 ksi (AISI 2002 Ex. III-6)



8.5-1 Flexural strength for a fully braced member

Determination of the bending strength for a fully braced member is equivalent to determining the effective section modulus at yield in the main *Specification*.

Finite strip analysis of 8ZS2.25x059 in pure bending:



Inputs from the finite strip analysis include:

$M_y := 107.53 \cdot \text{kip} \cdot \text{in}$

$M_{crl} := 0.85 \cdot M_y$ $\qquad M_{crl} = 91\ \text{kip} \cdot \text{in}$

$M_{crd} := 0.77 \cdot M_y$ $\qquad M_{crd} = 83\ \text{kip} \cdot \text{in}$

Since the member is assumed braced against lateral-torsional buckling, it is further assumed that bending about the x-axis is restrained, and thus $\sigma = M_y / I_x$ applies. The moment at which first yield occurs (M_y) is also determined based on this assumption.
If the member is free to twist then a moment about the x-axis must be resolved into moments about the principal 1 and 2 axes. Such a moment and the related analysis is shown in Section 3.2.5. However, this moment is not generally considered in cold-formed steel design of Z-sections since the applied moments are generally about the x-axis and to allow the twisting to occur would result in a highly inefficient cross-section. Thus, bracing is typically applied to restrain this twist.

Per DSM 1.2.2, M_n is the minimum of M_{ne}, $M_{n\ell}$, M_{nd}. For a fully braced member lateral-torsional buckling will not occur and thus $M_{ne} = M_y$, $M_{n\ell}$ and M_{nd} must still be checked.

$M_{ne} := M_y$ $\qquad M_{ne} = 108\ \text{kip} \cdot \text{in}$ $\qquad$ (fully braced)

Local buckling check per DSM 1.2.2.2

$$\lambda_l := \sqrt{\frac{M_{ne}}{M_{crl}}} \qquad \lambda_l = 1.08 \qquad \text{(subscript "l" = "}\ell\text{")} \qquad \text{(Eq. 1.2.2-7)}$$

(Continued) 8.5-1 Flexural strength for a fully braced member

Local buckling check per DSM 1.2.2.2 (continued)

$$M_{nl} := \begin{cases} M_{ne} & \text{if } \lambda_l \le 0.776 \\ \left[1 - 0.15\cdot\left(\frac{M_{crl}}{M_{ne}}\right)^{0.4}\right]\left(\frac{M_{crl}}{M_{ne}}\right)^{0.4}\cdot M_{ne} & \text{if } \lambda_l > 0.776 \end{cases}$$ (Eq. 1.2.2-5) (Eq. 1.2.2-6)

$M_{nl} = 87\,\text{kip}\cdot\text{in}$

Distortional buckling check per DSM 1.2.2.3

$\lambda_d := \sqrt{\frac{M_y}{M_{crd}}}$ $\lambda_d = 1.14$ (Eq. 1.2.2-10)

$$M_{nd} := \begin{cases} M_y & \text{if } \lambda_d \le 0.673 \\ \left[1 - 0.22\cdot\left(\frac{M_{crd}}{M_y}\right)^{0.5}\right]\left(\frac{M_{crd}}{M_y}\right)^{0.5}\cdot M_y & \text{if } \lambda_d > 0.673 \end{cases}$$ (Eq. 1.2.2-8) (Eq. 1.2.2-9)

$M_{nd} = 76\,\text{kip}\cdot\text{in}$ distortional buckling readily controls the strength of this section.

Predicted flexural strength per DSM 1.3

$M_n := \min((M_{ne}\ M_{nl}\ M_{nd}))$ $M_n = 76\,\text{kip}\cdot\text{in}$

The geometry of this cross-section falls within the "pre-qualified" beams of DSM 1.1.1.2 and the higher ϕ and lower Ω of DSM section 1.2.2 may therefore be used.

LRFD: $\phi_b := 0.9$ $\phi_b\cdot M_n = 69\,\text{kip}\cdot\text{in}$ flexural design strength

ASD: $\Omega_b := 1.67$ $\frac{M_n}{\Omega_b} = 46\,\text{kip}\cdot\text{in}$ flexural allowable design strength

Note on uplift

Part 4 of AISI (2002) *Design Manual* Example II-2 calculates the strength of this cross-section in uplift. The formula is RS_eF_y, where S_eF_y is the fully braced flexural strength and R is an empirical factor from main *Specification* C3.1.3. If the conditions of C3.1.3 are met, the strength in uplift can be found in this simple manner. For this example R=0.7.

uplift: $R := 0.7$ $R\cdot M_n = 53.3\,\text{kip}\cdot\text{in}$ nominal strength in uplift

The calibration of R factors was performed to the main *Specification*. The main *Specification* does not include an explicit check for distortional buckling. Therefore, if distortional buckling (M_{nd}) controls the strength, the uplift prediction using R may be excessively conservative. In this case M_{nd} does control the strength ($M_{n\ell}$ is considerably higher) thus this prediction is expected to be conservative. Arguably, a more accurate prediction would be $RM_{n\ell}$. As discussed in Section 3.3.7 of this Guide, it is possible to have the elastic buckling analysis include the influence of restraints such as deck - direct calculation instead of a reliance on empirical R factors would be the result.

8.5-2 Flexural strength for L=48.5 in. (AISI 2002 Example II-2)

AISI (2002) Example II-2 provides a complete calculation for a four-span continuous beam. The following calculation provides an alternative means to calculate the design bending strength of one of the spans. Namely, an interior span where L_y=L_t=48.5 in., and C_b = 1.67 (conservatively assumed as a linear moment diagram between the inflection point and support).

Inputs from the finite strip analysis include:

Local and distortional buckling values are unchanged from Example 8.5-1

For the interior span 8ZS2.5x059 at L=48.5 in., M_{cre}/M_y cannot be read directly from the finite strip analysis, because the section is experiencing distortional buckling at this length - see results in Section 3.2.5. If a curve is fit to the LTB range as discussed in Section 3.3.4 and detailed in Section 4.1, then M_{cre}/M_y is found to be 2.73 (ignoring moment gradient, i.e, for C_b=1).

$M_{cre} := 2.73 \cdot M_y$ for C_b=1 at L_y=L_t=48.5 in. $M_{cre} = 293.56\,\text{kip}\cdot\text{in}$

Lateral-torsional buckling check per DSM 1.2.2.1

per AISI (2002) Example II-2 $C_b := 1.67$

$M_{cre} := C_b \cdot M_{cre}$ $\frac{M_{cre}}{M_y} = 4.56$ per upperbounds of Section 2.2 M_{ne}=M_y! more formally,

$$M_{ne} := \begin{cases} M_{cre} & \text{if } M_{cre} < 0.56 \cdot M_y \quad \text{(Eq. 1.2.2-1)} \\ \frac{10}{9} \cdot M_y \cdot \left(1 - \frac{10 \cdot M_y}{36 \cdot M_{cre}}\right) & \text{if } 2.78 \cdot M_y \geq M_{cre} \geq 0.56 \cdot M_y \quad \text{(Eq. 1.2.2-2)} \\ M_y & \text{if } M_{cre} > 2.78 \cdot M_y \quad \text{(Eq. 1.2.2-3)} \end{cases}$$

$M_{ne} = 107.53\,\text{kip}\cdot\text{in}$ same as $M_y = 107.53\,\text{kip}\cdot\text{in}$

Local buckling check per DSM 1.2.2.2

unchanged from Example 8.5-1 $M_{nl} = 87\,\text{kip}\cdot\text{in}$

Distortional buckling check per DSM 1.2.2.3

unchanged from Example 8.5-1

$M_{nd} = 76\,\text{kip}\cdot\text{in}$ M_{nd} controls, at longer spans when M_{ne}<M_y, $M_{n\ell}$ may control.

Predicted flexural strength per DSM 1.3

$M_n := \min\left(\left(M_{ne}\ \ M_{nl}\ \ M_{nd}\right)\right)$ $M_n = 76\,\text{kip}\cdot\text{in}$ flexural nominal strength

The geometry of this cross-section falls within the "pre-qualified" beams of DSM Section 1.1.1.2 and the higher ϕ and lower Ω of DSM Section 1.2.2 may therefore be used.

LRFD: $\phi_b := 0.9$ $\phi_b \cdot M_n = 69\,\text{kip}\cdot\text{in}$ ASD: $\Omega_b := 1.67$ $\frac{M_n}{\Omega_b} = 46\,\text{kip}\cdot\text{in}$

Further notes on AISI (2002) Design Example II-2: Design checks for shear could follow the nomenclature of Section 4.4.1 of this Guide. Design checks for web crippling are unmodified from the AISI (2002) example. In the design checks for combined bending and shear, and combined bending and crippling of the AISI (2002) example, M_{nxo}/Ω could be replaced with the result of Example 8.5-1.

8.5-3 Compressive strength for a continuously braced column

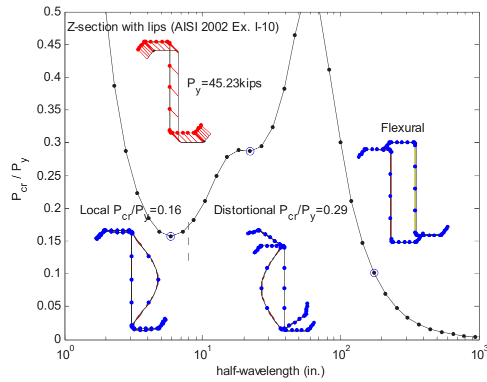
Finite strip analysis of 8ZS2.25x059 in compression is summarized in Section 3.2.5 and below.

Inputs from the finite strip analysis include:

$P_y := 45.23 \cdot kip$

$P_{crl} := 0.16 \cdot P_y$ $\quad P_{crl} = 7.2\,kip$

$P_{crd} := 0.29 \cdot P_y$ $\quad P_{crd} = 13.1\,kip$



Per DSM 1.2.1, P_n is the minimum of P_{ne}, $P_{n\ell}$, P_{nd}. If a column is continuously braced then global buckling P_{ne} is restricted and the squash load will develop if the section is compact.

$P_{ne} := P_y$ $\quad P_{ne} = 45.23\,kip$

Local buckling check per DSM 1.2.1.2

$$\lambda_l := \sqrt{\frac{P_{ne}}{P_{crl}}} \qquad \lambda_l = 2.5 \qquad \text{(subscript "l" = "}\ell\text{")} \qquad \text{(Eq. 1.2.1-7)}$$

$$P_{nl} := \begin{cases} P_{ne} & \text{if } \lambda_l \le 0.776 \\ \left[1 - 0.15 \cdot \left(\frac{P_{crl}}{P_{ne}}\right)^{0.4}\right]\left(\frac{P_{crl}}{P_{ne}}\right)^{0.4} \cdot P_{ne} & \text{if } \lambda_l > 0.776 \end{cases} \qquad \begin{matrix}\text{(Eq. 1.2.1-5)}\\ \text{(Eq. 1.2.1-6)}\end{matrix}$$

$P_{nl} = 20.16\,kip$

Distortional buckling check per DSM 1.2.1.3

$$\lambda_d := \sqrt{\frac{P_y}{P_{crd}}} \qquad \lambda_d = 1.86 \qquad \text{(Eq. 1.2.1-10)}$$

$$P_{nd} := \begin{cases} P_y & \text{if } \lambda_d \le 0.561 \\ \left[1 - 0.25 \cdot \left(\frac{P_{crd}}{P_y}\right)^{0.6}\right]\left(\frac{P_{crd}}{P_y}\right)^{0.6} \cdot P_y & \text{if } \lambda_d > 0.561 \end{cases} \qquad \begin{matrix}\text{(Eq. 1.2.1-8)}\\ \text{(Eq. 1.2.1-9)}\end{matrix}$$

$P_{nd} = 19\,kip$ $\quad$ though P_{crd} is significantly greater than $P_{cr\ell}$, P_{nd} still controls the strength, reflecting the reduced post-buckling reserve in distortional failures.

Predicted compressive strength per DSM 1.2

$P_n := \min\left(\left(P_{ne}\ \ P_{nl}\ \ P_{nd}\right)\right)$ $\quad P_n = 19\,kip$

The geometry of this cross-section falls within the "pre-qualified" columns of DSM 1.1.1.1 and the higher ϕ and lower Ω of DSM Section 1.2.1 may therefore be used.

LRFD: $\quad \phi_c := 0.85 \quad \phi_c \cdot P_n = 16.1\,kip$

ASD: $\quad \Omega_c := 1.80 \quad \frac{P_n}{\Omega_c} = 10.5\,kip$

8.5-4 Compression strength at F_n=25.9 ksi (AISI 2002 Example III-6)

AISI (2002) Example III-6 provides a complete example for the compressive strength of a 25 ft. long 8ZS2.25x059 with one flange through-fastened to deck or sheathing.

In AISI (2002) Example III-6 the solution for P_n considers (a) restrained buckling about a horizontal axis through the centroid and (b) torsional-flexural buckling following the empirical procedures outlined in the main *Specification* C4.6.

(a) Restrained buckling about an horizontal axis through the centroid

Flexural, torsional, or torsional-flexural check per DSM 1.2.1.1

Consider that due to the deck or sheathing the section is restricted so that it can only buckle about a horizontal axis through the centroid. This restriction could be imposed on the finite strip model, as described in Section 3.4.6. However, this is more cumbersome than simply calculating the elastic buckling strength using closed-form formulas. The strong axis elastic buckling stress may be found as detailed in AISI (2002) Example III-6, but involves nothing more than $F_e=\pi^2E/(KL_x/r_x)^2$ for this cross-section.

$F_e := 30.5\cdot ksi$ per AISI (2002) Example III-6

$P_{cre} := A_g \cdot F_e$ (note $A_g = 0.82\,in^2$)

$$\lambda_c := \sqrt{\frac{P_y}{P_{cre}}} \qquad \lambda_c = 1.34 \quad \text{(inelastic regime)} \qquad \text{(Eq. 1.2.1-1)}$$

$$P_{ne} := \begin{cases} 0.658^{\lambda_c^2} \cdot P_y & \text{if } \lambda_c \le 1.5 \quad \text{(Eq. 1.2.1-2)} \\ \frac{.877}{\lambda_c^2} \cdot P_y & \text{if } \lambda_c > 1.5 \quad \text{(Eq. 1.2.1-3)} \end{cases}$$

$P_{ne} = 21.26\,kip$

The stress associated with this load is $\frac{P_{ne}}{A_g} = 25.9\,ksi$ which is F_n in the main *Specification*.

As demonstrated in AISI (2002) Example I-10, in the main *Specification* effective area (A_e) calculations for columns are performed at the stress that the long column can maintain, this stress known as F_n is calculated per Eq. C4-2 or C4-3. A similar procedure is performed in the Direct Strength Method where the local buckling slenderness and strength equations (DSM Eqs. 1.2.1-7, and 1.2.1-5,6) are calculated at the long column strength, i.e., $P_{ne}=A_gF_n$.

(Continued) 8.5-4 Compressive strength at F_n=25.9 ksi (AISI 2002 Example III-6)

Local buckling check per DSM 1.2.1.2

$$\lambda_l := \sqrt{\frac{P_{ne}}{P_{crl}}} \qquad \lambda_l = 1.71$$ (compare with λ_ℓ of 2.5 for the same column with continuous bracing, see Example 8.5-2) (Eq. 1.2.1-7)

$$P_{nl} := \begin{cases} P_{ne} & \text{if } \lambda_l \le 0.776 \\ \left[1 - 0.15\cdot\left(\frac{P_{crl}}{P_{ne}}\right)^{0.4}\right]\left(\frac{P_{crl}}{P_{ne}}\right)^{0.4}\cdot P_{ne} & \text{if } \lambda_l > 0.776 \end{cases}$$ (Eq. 1.2.1-5) (Eq. 1.2.1-6)

$P_{nl} = 12.5\,\text{kip}$ (down from 20.2 kips for a column with continuous bracing)

Distortional buckling check per DSM 1.2.1.3

$$\lambda_d := \sqrt{\frac{P_y}{P_{crd}}} \qquad \lambda_d = 1.86$$ (Eq. 1.2.1-10)

$$P_{nd} := \begin{cases} P_y & \text{if } \lambda_d \le 0.561 \\ \left[1 - 0.25\cdot\left(\frac{P_{crd}}{P_y}\right)^{0.6}\right]\left(\frac{P_{crd}}{P_y}\right)^{0.6}\cdot P_y & \text{if } \lambda_d > 0.561 \end{cases}$$ (Eq. 1.2.1-8) (Eq. 1.2.1-9)

$P_{nd} = 19\,\text{kip}$ (note, the distortional buckling prediction is the same as Example 8.5-2, and now no longer controls the predicted strength)

Predicted compressive strength per DSM 1.2

$P_n := \min((P_{ne}\ \ P_{nl}\ \ P_{nd}))$ $\qquad P_n = 12.5\,\text{kip}$

The geometry of this cross-section falls within the "pre-qualified" columns of DSM 1.1.1.1 and the higher ϕ and lower Ω of DSM Section 1.2.1 may therefore be used.

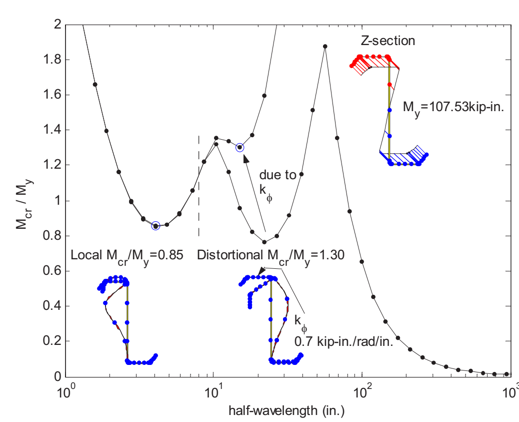
LRFD: $\phi_c := 0.85$ $\qquad \phi_c\cdot P_n = 10.6\,\text{kip}$

ASD: $\Omega_c := 1.80$ $\qquad \frac{P_n}{\Omega_c} = 6.9\,\text{kip}$

(b) Torsional-flexural buckling via the empirical procedures of main *Specification* C4.6

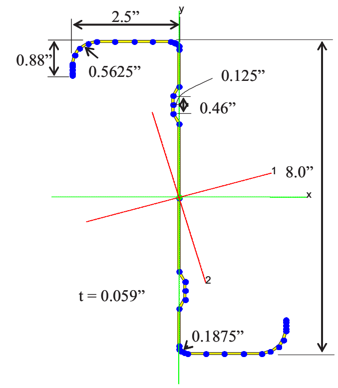
AISI (2002) Example III-6 part 2 provides the solution using the *Specification* equations of C4.6. The predicted capacity, P_n is 11.8 kips by this method.

The impact of partial restraint on buckling modes is discussed in Section 3.4.7. The finite strip analysis to the right shows a key aspect of this discussion - restraint of a flange in compression can have a significant positive impact on distortional buckling. If the stiffness of the deck or sheathing can be reliably known, then inclusion of its stiffness directly in the finite strip model is possible. Appropriate modeling for compression members is also possible, but not included here.



8.6 Z-section *modified*

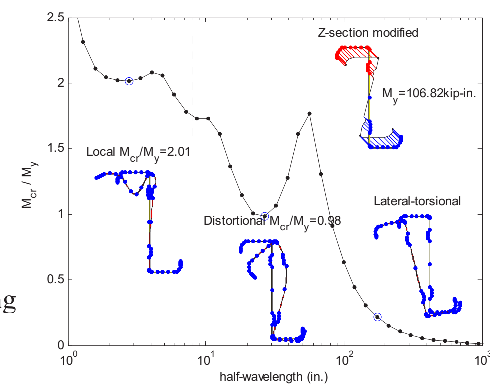
Given:
a. Steel: $F_y = 55$ ksi
b. Section 8ZS2.25x059 *modified* as shown to the right
c. Finite strip analysis results (Section 3.2.6)
Required:
1. Flexural strength for fully braced member
2. Compressive strength for a continuously braced column
3. Compressive strength at F_n=25.9 ksi



8.6-1 Flexural strength for a fully braced member

Determination of the bending capacity for a fully braced member is equivalent to determining the effective section modulus at yield in the main *Specification*.

Finite strip analysis of the modified Z-section:



Inputs from the finite strip analysis include:

$M_y := 106.82 \cdot \text{kip} \cdot \text{in}$

$M_{crl} := 2.01 \cdot M_y$ $\quad\quad M_{crl} = 215 \text{ kip} \cdot \text{in}$

$M_{crd} := 0.98 \cdot M_y$ $\quad\quad M_{crd} = 105 \text{ kip} \cdot \text{in}$

Since the member is assumed braced against lateral-torsional buckling, it is further assumed that bending about the x-axis is restrained bending, and thus $\sigma = M_y/I_x$ applies. The moment at which first yield occurs (M_y) is also determined based on this assumption.

Per DSM 1.2.2, M_n is the minimum of M_{ne}, $M_{n\ell}$, M_{nd}. For a fully braced member lateral-torsional buckling will not occur and thus $M_{ne} = M_y$, $M_{n\ell}$ and M_{nd} must still be checked.

$M_{ne} := M_y$ $\quad\quad M_{ne} = 107 \text{ kip} \cdot \text{in}$ $\quad$ (fully braced)

Local buckling check per DSM 1.2.2.2

$\lambda_1 := \sqrt{\dfrac{M_{ne}}{M_{crl}}}$ $\quad\quad \lambda_1 = 0.71$ $\quad\quad$ (subscript "l" = "ℓ") $\quad\quad$ (Eq. 1.2.2-7)

(Continued) 8.6-1 Flexural strength for a fully braced member

local buckling check per DSM 1.2.2.2 (continued)

$$M_{nl} := \begin{cases} M_{ne} & \text{if } \lambda_l \le 0.776 \\ \left[1 - 0.15\cdot\left(\frac{M_{crl}}{M_{ne}}\right)^{0.4}\right]\left(\frac{M_{crl}}{M_{ne}}\right)^{0.4}\cdot M_{ne} & \text{if } \lambda_l > 0.776 \end{cases}$$ (Eq. 1.2.2-5) (Eq. 1.2.2-6)

$M_{nl} = 107\ \text{kip}\cdot\text{in}$ The local buckling nominal strength is near the yield moment. While local buckling does not control the strength of this cross-section in the fully braced condition (distortional buckling does, see below) for discrete bracing at longer unbraced lengths local buckling in interaction with global buckling will control the strength. Thus, the primary benefit of this cross-section will be to provide higher capacities at longer unbraced lengths since little to no reduction will occur due to local buckling. See Chapter 4 for further results.

Distortional buckling check per DSM 1.2.2.3

$$\lambda_d := \sqrt{\frac{M_y}{M_{crd}}} \qquad \lambda_d = 1.01$$ (Eq. 1.2.2-10)

$$M_{nd} := \begin{cases} M_y & \text{if } \lambda_d \le 0.673 \\ \left[1 - 0.22\cdot\left(\frac{M_{crd}}{M_y}\right)^{0.5}\right]\left(\frac{M_{crd}}{M_y}\right)^{0.5}\cdot M_y & \text{if } \lambda_d > 0.673 \end{cases}$$ (Eq. 1.2.2-8) (Eq. 1.2.2-9)

$M_{nd} = 83\ \text{kip}\cdot\text{in}$ Distortional buckling readily controls the strength of this cross-section. The modified lip stiffener (from the original 8ZS2.25x059) increases the predicted nominal strength from 76 to 83 kip-in., or 9%.

Predicted flexural strength per DSM 1.3

$$M_n := \min\left(\left(M_{ne}\ \ M_{nl}\ \ M_{nd}\right)\right) \qquad M_n = 83\ \text{kip}\cdot\text{in}$$

The geometry of this cross-section is not "pre-qualified" (due to the web stiffeners) and thus the lower ϕ and higher Ω of the rational analysis clause, main *Specification* A1.1(b), are used.

LRFD: $\phi_b := 0.8 \qquad \phi_b\cdot M_n = 66\ \text{kip}\cdot\text{in}$

ASD: $\Omega_b := 2.0 \qquad \frac{M_n}{\Omega_b} = 41\ \text{kip}\cdot\text{in}$

8.6-2 Compressive strength for a continuously braced column

Finite strip analysis of the modified Z-section:

Inputs from the finite strip analysis include:

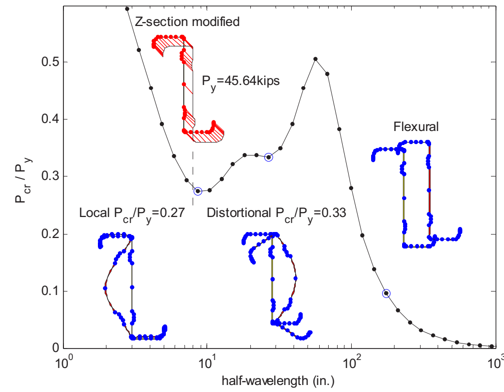
$P_y := 45.64 \cdot kip$

$P_{crl} := 0.27 \cdot P_y$ $\quad P_{crl} = 12.3\ kip$

$P_{crd} := 0.33 \cdot P_y$ $\quad P_{crd} = 15.1\ kip$

Per DSM 1.2.1, P_n is the minimum of P_{ne}, $P_{n\ell}$, P_{nd}. If a column is continuously braced then global buckling P_{ne} is restricted and the squash load will develop if the cross-section is compact.

$P_{ne} := P_y$ $\quad P_{ne} = 45.64\ kip$



Local buckling check per DSM 1.2.1.2

$$\lambda_l := \sqrt{\frac{P_{ne}}{P_{crl}}} \qquad \lambda_l = 1.92 \qquad \text{(subscript "l" = "}\ell\text{")} \qquad \text{(Eq. 1.2.1-7)}$$

$$P_{nl} := \begin{cases} P_{ne} & \text{if } \lambda_l \le 0.776 \\ \left[1 - 0.15 \cdot \left(\frac{P_{crl}}{P_{ne}}\right)^{0.4}\right]\left(\frac{P_{crl}}{P_{ne}}\right)^{0.4} \cdot P_{ne} & \text{if } \lambda_l > 0.776 \end{cases} \qquad \begin{matrix}\text{(Eq. 1.2.1-5)}\\ \text{(Eq. 1.2.1-6)}\end{matrix}$$

$P_{nl} = 24.63\ kip$

Distortional buckling check per DSM 1.2.1.3

$$\lambda_d := \sqrt{\frac{P_y}{P_{crd}}} \qquad \lambda_d = 1.74 \qquad \text{(Eq. 1.2.1-10)}$$

$$P_{nd} := \begin{cases} P_y & \text{if } \lambda_d \le 0.561 \\ \left[1 - 0.25 \cdot \left(\frac{P_{crd}}{P_y}\right)^{0.6}\right]\left(\frac{P_{crd}}{P_y}\right)^{0.6} \cdot P_y & \text{if } \lambda_d > 0.561 \end{cases} \qquad \begin{matrix}\text{(Eq. 1.2.1-8)}\\ \text{(Eq. 1.2.1-9)}\end{matrix}$$

$P_{nd} = 20.5\ kip$ though P_{crd} is greater than $P_{cr\ell}$, P_{nd} still controls the strength, reflecting the reduced post-buckling reserve in distortional buckling failures.

Predicted compressive strength per DSM 1.2

$P_n := \min\left(\left(P_{ne}\ \ P_{nl}\ \ P_{nd}\right)\right)$ $\quad P_n = 20.5\ kip$

The geometry of this section falls outside the "pre-qualified" columns of DSM 1.1.1.1 and thus the lower ϕ and higher Ω of main *Specification* A1.1(b) are used.

LRFD: $\phi_c := 0.8$ $\quad \phi_c \cdot P_n = 16.4\ kip$

ASD: $\Omega_c := 2.00$ $\quad \frac{P_n}{\Omega_c} = 10.2\ kip$

8.6-3 Compressive strength at F_n=25.9 ksi

As discussed in Example 8.5-4, AISI (2002) Example III-6 considers the compressive strength of a Z attached to deck or sheathing. Two failure modes are considered, the first of which is restrained flexural buckling about a horizontal axis through the centroid. For the 8ZS2.25x059 the inelastic buckling stress in this mode, F_n, is 25.9 ksi. In the main *Specification* effective area (A_e) calculations for columns are performed at the stress that the long column can maintain, this stress known as F_n is calculated per Eq. C4-2 or C4-3. A similar procedure is performed in the Direct Strength Method where the local buckling slenderness and strength equations (DSM Eq.'s 1.2.1-7, and 1.2.1-5,6) are calculated at the strength a long column can maintain, i.e., $P_{ne}=A_gF_n$.

for this example, F_n is assumed at 25.9 ksi, therefore, (note $A_g = 0.83\ in^2$)

$P_{ne} := A_g \cdot 25.9 \cdot ksi$ $\quad$ $P_{ne} = 21.49\ kip$

Local buckling check per DSM 1.2.1.2

$\lambda_l := \sqrt{\dfrac{P_{ne}}{P_{crl}}}$ $\quad$ $\lambda_l = 1.32$ $\quad$ (compare with λ_ℓ of 1.9 for the same column with continuous bracing, see 3.2.5-2) (Eq. 1.2.1-7)

$$P_{nl} := \begin{cases} P_{ne} & \text{if } \lambda_l \le 0.776 \\ \left[1 - 0.15\cdot\left(\dfrac{P_{crl}}{P_{ne}}\right)^{0.4}\right]\left(\dfrac{P_{crl}}{P_{ne}}\right)^{0.4}\cdot P_{ne} & \text{if } \lambda_l > 0.776 \end{cases}$$

(Eq. 1.2.1-5)
(Eq. 1.2.1-6)

$P_{nl} = 15.1\ kip$ (down from 24.6 kips for a column with continuous bracing)

Distortional buckling check per DSM 1.2.1.3

$\lambda_d := \sqrt{\dfrac{P_y}{P_{crd}}}$ $\quad$ $\lambda_d = 1.74$ (Eq. 1.2.1-10)

$$P_{nd} := \begin{cases} P_y & \text{if } \lambda_d \le 0.561 \\ \left[1 - 0.25\cdot\left(\dfrac{P_{crd}}{P_y}\right)^{0.6}\right]\left(\dfrac{P_{crd}}{P_y}\right)^{0.6}\cdot P_y & \text{if } \lambda_d > 0.561 \end{cases}$$

(Eq. 1.2.1-8)
(Eq. 1.2.1-9)

$P_{nd} = 20.5\ kip$ (note, the distortional buckling prediction is the same as Example 8.6-2, but now no longer controls the predicted strength)

Predicted compressive strength per DSM 1.2

$P_n := \min\left(\left(P_{ne}\ \ P_{nl}\ \ P_{nd}\right)\right)$ $\quad$ $P_n = 15.1\ kip$

The geometry of this section falls outside the "pre-qualified" columns of DSM 1.1.1.1 and the lower ϕ and higher Ω of main *Specification* A1.1(b) are used.

LRFD: $\phi_c := 0.80$ $\quad$ $\phi_c \cdot P_n = 12.11\ kip$

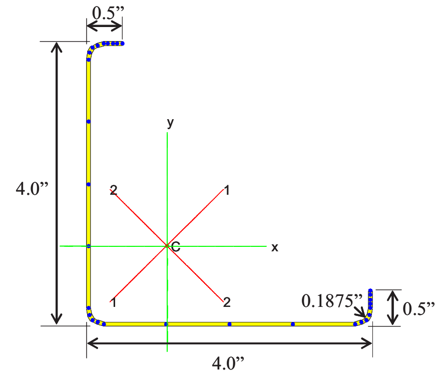
ASD: $\Omega_c := 2.00$ $\quad$ $\dfrac{P_n}{\Omega_c} = 7.6\ kip$

(note, the modified cross-section provides greater strength than the unmodified Z-section of Example 8.5-4)

Note, see Example 8.5-4 for additional discussion on the torsional-flexural compressive strength of a cross-section with decking or sheathing attached to one flange.

8.7 Equal leg angle with lips

Given:
a. Steel: $F_y = 50$ ksi
b. Section 4LS4x060 as shown to the right
c. Finite strip analysis results (Section 3.2.7)
Required:
1. Flexural strength about x-axis for a fully braced member
2. Flexural strength about minimum principal axis for L=18 in. (AISI 2002 Example III-4)
3. Compressive strength for a continuously braced member
4. Compressive strength at F_n=14.7 ksi (AISI 2002 Example III-4)
5. Compression strength considering eccentricity (AISI 2002 Example III-4)

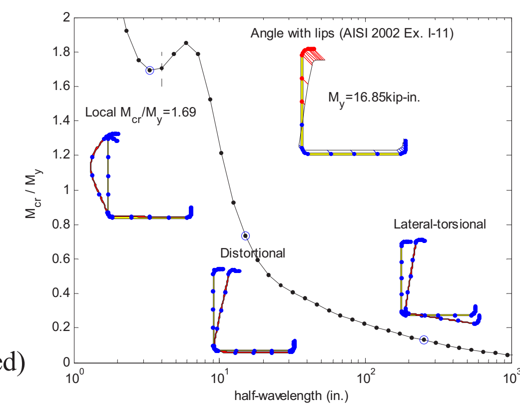


8.7-1 Flexural strength for a fully braced member

Determination of the bending strength for a fully braced member is equivalent to determining the effective section modulus at yield in the main *Specification*.

Consider finite strip analysis of 4LS4x060 in restrained bending as summarized in Section 3.2.7.

Inputs from the finite strip analysis include:



$M_y := 16.85 \cdot \text{kip} \cdot \text{in}$

$M_{crl} := 1.69 \cdot M_y \qquad M_{crl} = 28\,\text{kip} \cdot \text{in}$

Per DSM 1.2.2, M_n is the minimum of M_{ne}, $M_{n\ell}$, M_{nd}. For a fully braced member lateral-torsional buckling will not occur and thus $M_{ne} = M_y$, $M_{n\ell}$ and M_{nd} must still be checked.

$M_{ne} := M_y \qquad M_{ne} = 17\,\text{kip} \cdot \text{in}$ (fully braced)

Local buckling check per DSM 1.2.2.2

Per Section 2.2 of this Guide $M_{cr\ell}$>1.66M_y, therefore $M_{nl} := M_y$

Distortional buckling check per DSM 1.2.2.3

The distortional buckling check depends on the unbraced length. In the FSM analysis the largest unbraced length at which the distortional buckling is observed is about 20 in. M_{crd}/M_y is about 0.6 at this length. Thus, this value could be conservatively used for distortional buckling for any length longer than 20 in. However, if the member is braced against LTB, in this example it is assumed the member is braced against distortional buckling as well. Therefore, for a fully braced member it is expected that no reduction occurs due to distortional buckling and M_{nd}=M_y.

Predicted flexural strength per DSM 1.3

$M_n := \min\left(\left(M_{ne} \;\; M_{nl} \;\; M_{nd}\right)\right) \qquad M_n = 17\,\text{kip} \cdot \text{in}$

The geometry of this section falls outside the "pre-qualified" beams and the lower ϕ and higher Ω of the main *Specification* (Section A1.1b) must be used.

LRFD: $\phi_b := 0.8 \quad \phi_b \cdot M_n = 13\,\text{kip} \cdot \text{in}$ ASD: $\Omega_b := 2.0 \quad \dfrac{M_n}{\Omega_b} = 8\,\text{kip} \cdot \text{in}$

8.7-2 Flexural strength about minimum principal axis for L=18 in., (AISI 2002 Example III-4)

Example III-4 of AISI (2002) consider the compressive strength of this cross-section over an 18 in. length, however, per Sections C4(b) and C5.2 of the main *Specification*, the effect of an eccentricity of PL/1000 about the minor axis must be considered.

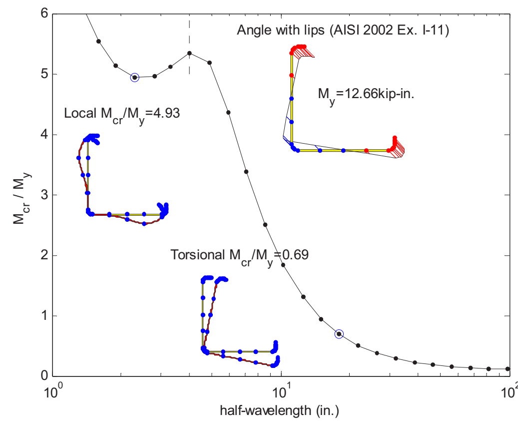
This example provides the bending strength about the minimum principal axis at L=18 in.

Inputs from the finite strip analysis include:

$M_y := 12.66 \cdot kip \cdot in$

$M_{crl} := 4.93 \cdot M_y$ $\qquad M_{crl} = 62\, kip \cdot in$

$M_{cre} := 0.69 \cdot M_y$ at 18 in. $\qquad M_{cre} = 8.74\, kip \cdot in$



Lateral-torsional buckling check per DSM 1.2.2.1

$$M_{ne} := \begin{cases} M_{cre} \text{ if } M_{cre} < 0.56 \cdot M_y & \text{(Eq. 1.2.2-1)} \\ \frac{10}{9} \cdot M_y \cdot \left(1 - \frac{10 \cdot M_y}{36 \cdot M_{cre}}\right) \text{ if } 2.78 \cdot M_y \geq M_{cre} \geq 0.56 \cdot M_y & \text{(Eq. 1.2.2-2)} \\ M_y \text{ if } M_{cre} > 2.78 \cdot M_y & \text{(Eq. 1.2.2-3)} \end{cases}$$

$M_{ne} = 8.4\, kip \cdot in$ $\qquad \frac{M_{ne}}{S_{g22}} = 33.19\, ksi$ that can be compared with an F_c of 33.47 ksi calculated in AISI (2002) Example III-4.

Local buckling check per DSM 1.2.2.2

$M_{cr\ell}$ is significantly greater than $1.66M_y$ and therefore no local reduction will occur.

$M_{nl} := M_{ne}$

Distortional buckling check per DSM 1.2.2.3

M_{crd} is not relevant to this cross-section as it is not separate from the global (lateral-torsional) mode, set M_{nd} to M_y in this case.

$M_{nd} := M_y$

Predicted flexural strength per DSM 1.3

$M_n := \min\left(\left(M_{ne} \;\; M_{nl} \;\; M_{nd}\right)\right)$ $\qquad M_n = 8.4\, kip \cdot in$

The geometry of this section falls outside the "pre-qualified" beams and the lower ϕ and higher Ω of the main *Specification* (Section A1.1(b)) must be used.

LRFD: $\phi_b := 0.8$ $\quad \phi_b \cdot M_n = 6.72\, kip \cdot in$ $\qquad$ ASD: $\Omega_b := 2.0$ $\quad \frac{M_n}{\Omega_b} = 4.2\, kip \cdot in$

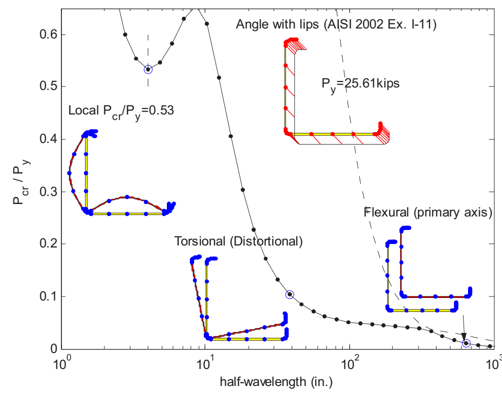
8.7-3 Compressive strength for a continuously braced column

Finite strip analysis of 4LS4x060 in pure compression is summarized in Section 3.2.7.

Inputs from the finite strip analysis include:

$P_y := 25.61 \cdot kip$

$P_{crl} := 0.53 \cdot P_y$ $\qquad P_{crl} = 13.6\,kip$



Per DSM 1.2.1, P_n is the minimum of P_{ne}, $P_{n\ell}$, P_{nd}. If a column is continuously braced then global buckling P_{ne} is restricted and the squash load will develop if the section is compact.

$P_{ne} := P_y$ $\qquad P_{ne} = 25.61\,kip$

Local buckling check per DSM 1.2.1.2

$$\lambda_l := \sqrt{\frac{P_{ne}}{P_{crl}}} \qquad \lambda_l = 1.37 \qquad \text{(subscript "l" = "}\ell\text{")} \qquad \text{(Eq. 1.2.1-7)}$$

$$P_{nl} := \begin{cases} P_{ne} & \text{if } \lambda_l \le 0.776 \\ \left[1 - 0.15 \cdot \left(\frac{P_{crl}}{P_{ne}}\right)^{0.4}\right]\left(\frac{P_{crl}}{P_{ne}}\right)^{0.4} \cdot P_{ne} & \text{if } \lambda_l > 0.776 \end{cases} \qquad \begin{matrix}\text{(Eq. 1.2.1-5)} \\ \text{(Eq. 1.2.1-6)}\end{matrix}$$

$P_{nl} = 17.55\,kip$

Distortional buckling check per DSM 1.2.1.3

Distortional buckling is not distinct from torsional buckling for this cross-section. If this cross-section is fully braced then distortional buckling is irrelevant to the design strength, the limit state may be ignored or equivalently, set $P_{nd}=P_y$.

Predicted compressive strength per DSM 1.2

$P_n := \min((P_{ne}\ \ P_{nl}\ \ P_{nd}))$ $\qquad P_n = 17.6\,kip$

The geometry of this section falls outside the "pre-qualified" columns of DSM 1.1.1.1 and the lower ϕ and higher Ω of the rational analysis clause in the main *Specification* (A1.1(b)) is used:

LRFD: $\quad \phi_c := 0.8 \qquad \phi_c \cdot P_n = 14\,kip$

ASD: $\quad \Omega_c := 2.00 \qquad \frac{P_n}{\Omega_c} = 8.8\,kip$

8.7-4 Compressive strength at F_n=14.7 ksi (AISI 2002 Example III-4)

AISI (2002) Example III-4 considers this 4LS4x060 cross-section as an 18 in. long column. The cross-section is concentrically loaded in compression and KL_x=KL_y=KL_t=18 in. AISI (2002) Example III-4 provides the lengthy *Specification* formulas (C4) for determining the global buckling strength of this column, while AISI (2002) Example I-4 provides the effective area. This example provides the concentric compressive strength using DSM. Finite strip analysis of 4LS4x060 in pure compression is the same as used in Example 8.7-3 with P_y, $P_{cr\ell}$, etc., from the FSM results (Section 3.2.7), the following is obtained:

$P_{cre} := 0.31 \cdot P_y$ at 18 in. $P_{cre} = 7.94\,\text{kip}$ $\frac{P_{cre}}{A_g} = 15.5\,\text{ksi}$ compare with 16.7 ksi by the formula given in Example III-4 of AISI (2002)

Flexural, torsional, or torsional-flexural check per DSM 1.2.1.1

$\lambda_c := \sqrt{\frac{P_y}{P_{cre}}}$ $\lambda_c = 1.8$ (in the elastic buckling regime) (Eq. 1.2.1-1)

$$P_{ne} := \begin{cases} 0.658^{\lambda_c^2} \cdot P_y & \text{if } \lambda_c \le 1.5 \quad \text{(Eq. 1.2.1-2)} \\ \frac{.877}{\lambda_c^2} \cdot P_y & \text{if } \lambda_c > 1.5 \quad \text{(Eq. 1.2.1-3)} \end{cases}$$

$P_{ne} = 6.96\,\text{kip}$ or as stress: $\frac{P_{ne}}{A_g} = 13.59\,\text{ksi}$ which itself is F_n in the main *Specification* (14.7 ksi in example III-4 of AISI 2002).

In the main *Specification* effective area (A_e) calculations for columns are performed at the stress that a long column can maintain, this stress known as F_n is calculated per Eq. C4-2 or C4-3. A similar procedure is performed in the Direct Strength Method where the local buckling slenderness and strength equations (1.2.1-7, and 1.2.1-5,6) are calculated at the strength a long column can maintain, i.e., P_{ne}=A_gF_n. Per DSM 1.2.1, P_n is the minimum of P_{ne}, $P_{n\ell}$, P_{nd}.

for comparison to AISI (2002) Examples I-11 and III-4 assume F_n is 14.7 ksi and continue with the Direct Strength Method. (Alternatively P_{ne} calculated above could be used.)

$P_{ne} := A_g \cdot 14.7 \cdot \text{ksi}$ $P_{ne} = 7.53\,\text{kip}$

Local buckling check per DSM 1.2.1.2

$\lambda_l := \sqrt{\frac{P_{ne}}{P_{crl}}}$ $\lambda_l = 0.74$ (compare with λ_ℓ of 2.9 for the same column with continuous bracing, see Example 8.7-3) (Eq. 1.2.1-7)

since λ_ℓ<0.776 per Eq. 1.2.1-5 $P_{nl} := P_{ne}$ (down from 17.55 kips for continuous bracing)

Distortional buckling check per DSM 1.2.1.3

Distortional buckling is not distinct from torsional buckling in this section. The limit state may be ignored or equivalently, set P_{nd}=P_y.

Predicted compressive strength per DSM 1.2

$P_n := \min((P_{ne}\ \ P_{nl}\ \ P_{nd}))$ $P_n = 7.53\,\text{kip}$

The geometry of this section falls outside the "pre-qualified" columns of DSM 1.1.1.1 and the lower ϕ and higher Ω of the rational analysis clause in the main *Specification* (A1.1(b)) is used:

LRFD: $\phi_c := 0.8$ $\phi_c \cdot P_n = 6.02\,\text{kip}$ ASD: $\Omega_c := 2.00$ $\frac{P_n}{\Omega_c} = 3.76\,\text{kip}$

8.7-5 Compressive strength considering eccentricity (AISI 2002 Example III-4)

Continuing from Example 8.7-4 above, this example considers the compressive capacity of this same cross-section with the effect of an eccentricity of PL/1000 about the minor axis. This is a continuation of AISI (2002) *Design Manual* Example III-4.

Compression: the compressive strength of this cross-section as determined in Example 8.7-4.

$\phi_c = 0.8 \qquad P_n = 7.53\,\text{kip}$

for the C5.2.2-2 interaction equation the fully braced compressive strength is needed, per 8.7-3

$P_{no} = 17.6\,\text{kip}$

Bending: the flexural strength of this section as determined in Example 8.7-2 above

$\phi_b = 0.8 \qquad M_n = 8.4\,\text{kip}\cdot\text{in}$

Factors to account for approximate 2nd order analysis

$M_{ux}(P) := \dfrac{P\cdot L}{1000}$ First order required moment for L=18 in. is 0.0180P

$C_m := 1.0$ The member is pinned, accidental eccentricity places a constant required moment, 2nd order moments and primary moments are at the same location, and C_m should be 1.0.

$\alpha_2(P_u) := 1 - \dfrac{P_u}{P_{E2}}$ α_2 is the moment amplification term for minor principal axis bending moment. The demand axial load P is to be solved for in this case. The elastic buckling load about the minor principal axis can be determined by Equation C5.2.1-7 in the main *Specification*, or taken from finite strip analysis.

$P_{E2} := 357\cdot\text{kip}$ from Eq. C5.2.1-7 as used in AISI (2002) Example III-4

Interaction equations

assume $\dfrac{P_u}{\phi_c\cdot P_n}$ is > 0.15, therefore use Equations C5.2.2-1 and C5.2.2-2 and find P_u

$$\frac{P_u}{\phi_c\cdot P_n} + \frac{C_m\cdot M_u}{\phi_b\cdot M_n\cdot \alpha_2} \le 1 \qquad \frac{P_u}{0.8\cdot 7.53} + \frac{1.0\cdot 0.0180\cdot P_u}{0.8\cdot 8.4\cdot\left(1 - \dfrac{P_u}{357}\right)} \le 1 \qquad \text{(Eq. C5.2.2-1)}$$

$P_u \le 5.93$

$$\frac{P_u}{\phi_c\cdot P_{no}} + \frac{M_u}{\phi_b\cdot M_n} \le 1 \qquad \frac{P_u}{0.8\cdot 17.6} + \frac{0.0180\cdot P_u}{0.8\cdot 8.4} \le 1 \qquad \text{(Eq. C5.2.2-2)}$$

$P_u \le 13.57$

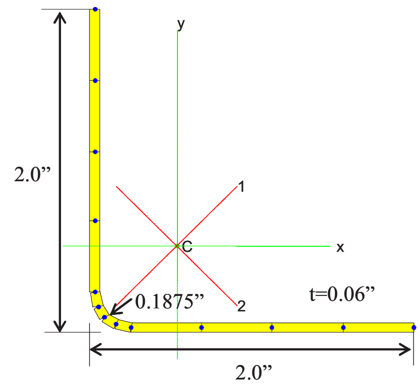
Per main *Specification* C5.2.2-1 the required strength (LRFD) P_u, is $5.93\cdot\text{kip}$

Check that $\dfrac{P_u}{\phi_c\cdot P_n}$ is greater than 0.15, $\dfrac{5.93}{0.8\cdot 7.53} = 0.98$ OK

See AISI (2002) Design Manual Example III-4 for ASD format.

8.8 Equal leg angle

Given:
a. Steel: $F_y = 33$ ksi
b. Section 2LU2x060 as shown to the right
c. Finite strip analysis results (Section 3.2.8)
Required:
1. Flexural strength for a fully braced member
2. Compressive strength for a continuously braced column
3. Compressive strength at F_n=12.0 ksi



8.8-1 Flexural strength for a fully braced member

Determination of the bending strength for a fully braced member is equivalent to determining the effective section modulus at yield in the main *Specification*.

Finite strip analysis of 2LU2x060 in restrained bending is summarized in Section 3.2.8 and below.

Inputs from the finite strip analysis include:

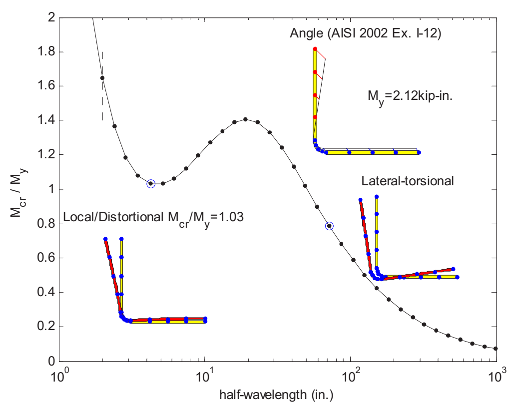
$M_y := 2.12 \cdot kip \cdot in$

$M_{crl} := 1.03 \cdot M_y$ $\qquad M_{crl} = 2.18\ kip \cdot in$

$M_{crd} := 1.03 \cdot M_y$ $\qquad M_{crd} = 2.18\ kip \cdot in$

Conservatively assume the observed mode could be either local or distortional for the angle.

Per DSM 1.2.2, M_n is the minimum of M_{ne}, $M_{n\ell}$, M_{nd}. For a fully braced member lateral-torsional buckling will not occur and thus $M_{ne} = M_y$, $M_{n\ell}$ and M_{nd} must still be checked.



$M_{ne} := M_y$ $\qquad M_{ne} = 2.12\ kip \cdot in$ $\qquad$ (fully braced)

Local buckling check per DSM 1.2.2.2

$$\lambda_1 := \sqrt{\frac{M_{ne}}{M_{crl}}} \qquad \lambda_1 = 0.99 \qquad \text{(subscript "1" = "}\ell\text{")} \qquad \text{(Eq. 1.2.2-7)}$$

$$M_{nl} := \begin{cases} M_{ne} \ \text{ if } \lambda_1 \le 0.776 & \text{(Eq. 1.2.2-5)} \\ \left[1 - 0.15 \cdot \left(\frac{M_{crl}}{M_{ne}}\right)^{0.4}\right]\left(\frac{M_{crl}}{M_{ne}}\right)^{0.4} \cdot M_{ne} \ \text{ if } \lambda_1 > 0.776 & \text{(Eq. 1.2.2-6)} \end{cases}$$

$M_{nl} = 1.82\ kip \cdot in$

Distortional buckling check per DSM 1.2.2.3

$\lambda_d := \sqrt{\frac{M_y}{M_{crd}}}$ $\lambda_d = 0.99$ (Eq. 1.2.2-10)

$M_{nd} := \begin{cases} M_y & \text{if } \lambda_d \le 0.673 \\ \left[1 - 0.22 \cdot \left(\frac{M_{crd}}{M_y}\right)^{0.5}\right]\left(\frac{M_{crd}}{M_y}\right)^{0.5} \cdot M_y & \text{if } \lambda_d > 0.673 \end{cases}$ (Eq. 1.2.2-8) (Eq. 1.2.2-9)

$M_{nd} = 1.67\,\text{kip}\cdot\text{in}$ Distortional buckling controls the predicted strength, which given $M_{cr\ell}$=M_{crd} will always be the case when LTB is assumed fully braced (M_{ne}=M_y), if LTB reduces the bending strength slightly M_{ne}<M_y, then $M_{n\ell}$ will quickly control the strength, for more on this topic see Chapter 4 of this Guide.

Predicted flexural strength per DSM 1.3

$M_n := \min((M_{ne}\ \ M_{nl}\ \ M_{nd}))$ $M_n = 1.67\,\text{kip}\cdot\text{in}$

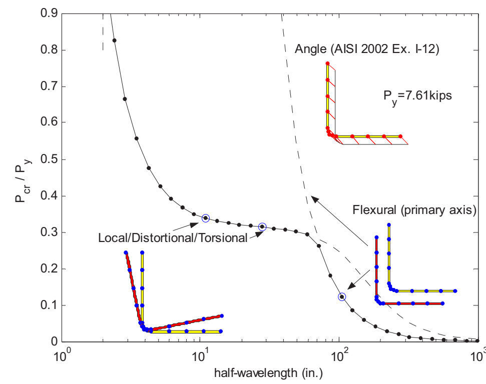
The geometry of this section falls outside the "pre-qualified" beams and the lower ϕ and higher Ω of the main *Specification* (Section A1.1(b)) must be used.

LRFD: $\phi_b := 0.8$ $\phi_b \cdot M_n = 1.34\,\text{kip}\cdot\text{in}$ ASD: $\Omega_b := 2.0$ $\frac{M_n}{\Omega_b} = 0.84\,\text{kip}\cdot\text{in}$

8.8-2 Compressive strength for a continuously braced column

Here the definition of "continuous" bracing plays an important role in determining how to proceed with the strength calculation. If the bracing restrains torsion and flexure then a specific analysis with the bracing included would be needed to determine what manner of local buckling may still occur. If the bracing restricts flexure only, but allows torsion then the plateau at approximately $0.3P_y$ would be relevant as a local torsional mode is possible. For this example, it is assumed that the bracing removes the long column flexural mode, but not the shorter torsional mode.

Finite strip analysis of 2LS2x060:



Inputs from the finite strip analysis include:

$P_y := 7.61 \cdot kip$

$P_{crl} := 0.3 \cdot P_y \qquad P_{crl} = 2.3\,kip$

Per DSM 1.2.1, P_n is the minimum of P_{ne}, $P_{n\ell}$, P_{nd}. If a column is continuously braced then global buckling P_{ne} is restricted and the squash load will develop if the cross-section is compact.

$P_{ne} := P_y \qquad P_{ne} = 7.61\,kip$

Local buckling check per DSM 1.2.1.2

$$\lambda_l := \sqrt{\frac{P_{ne}}{P_{crl}}} \qquad \lambda_l = 1.83 \qquad \text{(subscript "l" = "}\ell\text{")} \qquad \text{(Eq. 1.2.1-7)}$$

$$P_{nl} := \begin{cases} P_{ne} & \text{if } \lambda_l \le 0.776 \\ \left[1 - 0.15 \cdot \left(\frac{P_{crl}}{P_{ne}}\right)^{0.4}\right]\left(\frac{P_{crl}}{P_{ne}}\right)^{0.4} \cdot P_{ne} & \text{if } \lambda_l > 0.776 \end{cases} \qquad \begin{array}{l}\text{(Eq. 1.2.1-5)}\\ \text{(Eq. 1.2.1-6)}\end{array}$$

$P_{nl} = 4.27\,kip$

Distortional buckling check per DSM 1.2.1.3

Distortional buckling is not distinct from torsional buckling in this section. If this section is fully braced then distortional buckling is irrelevant to the design strength, we may ignore the limit state or equivalently, set P_{nd}=P_y.

Predicted compressive strength per DSM 1.2

$P_n := \min\left(\left(P_{ne} \;\; P_{nl} \;\; P_{nd}\right)\right) \qquad P_n = 4.3\,kip$

The geometry of this section falls outside the "pre-qualified" columns of DSM 1.1.1.1 and the lower ϕ and higher Ω of the rational analysis clause in the main *Specification* (A1.1(b)) is used:

LRFD: $\phi_c := 0.8 \qquad \phi_c \cdot P_n = 3.4\,kip$

ASD: $\Omega_c := 2.00 \qquad \frac{P_n}{\Omega_c} = 2.1\,kip$

8.8-3 Compression strength at F_n=12.0 ksi

Finite strip analysis of 2LU2x060 in pure compression is the same as used in Example 8.8-2.

In the main *Specification* effective area (A_e) calculations for columns are performed at the stress that a long column can maintain, this stress known as F_n is calculated per Eq. C4-2 or C4-3. A similar procedure is performed in the Direct Strength Method where the local buckling slenderness and strength equations (1.2.1-7, and 1.2.1-5,6) are calculated at the strength a long column can maintain, i.e., $P_{ne}=A_gF_n$. Per DSM 1.2.1, P_n is the minimum of P_{ne}, $P_{n\ell}$, P_{nd}.

$P_{ne} := A_g \cdot 12 \cdot ksi \qquad P_{ne} = 2.77\,kip$

From the finite strip analysis the long column buckling mode is likely flexure, in which case a local mode dominated by torsion may possibly interact. Conservatively, this interaction is included in the calculation.

Local buckling check per DSM 1.2.1.2

$\lambda_l := \sqrt{\dfrac{P_{ne}}{P_{crl}}} \qquad \lambda_l = 1.1$ (compare with λ_ℓ of 1.8 for the same column with continuous bracing, see Example 8.8-2) (Eq. 1.2.1-7)

$$P_{nl} := \begin{cases} P_{ne} & \text{if } \lambda_l \le 0.776 \\ \left[1 - 0.15\cdot\left(\dfrac{P_{crl}}{P_{ne}}\right)^{0.4}\right]\left(\dfrac{P_{crl}}{P_{ne}}\right)^{0.4}\cdot P_{ne} & \text{if } \lambda_l > 0.776 \end{cases}$$ (Eq. 1.2.1-5) (Eq. 1.2.1-6)

$P_{nl} = 2.21\,kip$ (down from 4.3 kips for a column with continuous bracing)

Distortional buckling check per DSM 1.2.1.3

Distortional buckling is not distinct from torsional buckling in this cross-section. If this cross-section is fully braced then distortional buckling is irrelevant to the design strength, the limit state may be ignored or equivalently, set $P_{nd}=P_y$.

Predicted compressive strength per DSM 1.2

$P_n := \min\left(\left(P_{ne}\ \ P_{nl}\ \ P_{nd}\right)\right) \qquad P_n = 2.21\,kip$

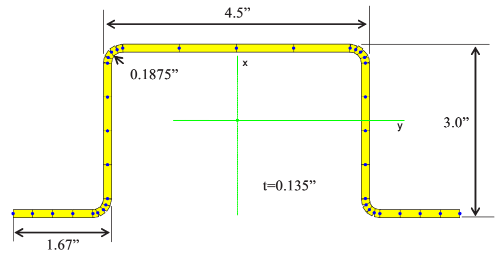
The geometry of this section falls outside the "pre-qualified" columns of DSM 1.1.1.1 and the lower ϕ and higher Ω of the rational analysis clause in the main *Specification* (A1.1(b)) is used:

LRFD: $\phi_c := 0.8 \qquad \phi_c \cdot P_n = 1.77\,kip$

ASD: $\Omega_c := 2.00 \qquad \dfrac{P_n}{\Omega_c} = 1.1\,kip$

8.9 Hat section

Given:
a. Steel: $F_y = 50$ ksi
b. Section 3HU4.5x135 as shown to the right
c. Finite strip analysis results (Section 3.2.9)
Required:
1. Flexural strength for a fully braced member (AISI 2002 Example II-4)
2. Compressive strength for a continuously braced column
3. Compressive strength for L=6 ft (AISI 2002 Example III-7)
4. Beam-column allowable strength (AISI 2002 Ex. III-7)



8.9-1 Flexural strength for a fully braced member (AISI 2002 Example II-4)

Determination of the bending strength for a fully braced member is equivalent to determining the effective section modulus at yield in the main *Specification*.

Finite strip analysis of 3HU4.5x135 in pure bending is summarized in Section 3.2.9, and below.

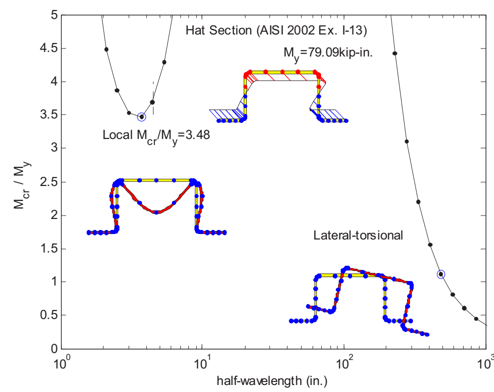
Inputs from the finite strip analysis include:

$M_y := 79.09 \cdot \text{kip} \cdot \text{in}$

$M_{crl} := 3.48 \cdot M_y \qquad M_{crl} = 275\ \text{kip} \cdot \text{in}$

Per DSM 1.2.2, M_n is the minimum of M_{ne}, $M_{n\ell}$, M_{nd}. For a fully braced member lateral-torsional buckling will not occur and thus $M_{ne} = M_y$

$M_{ne} := M_y \qquad M_{ne} = 79\ \text{kip} \cdot \text{in}$ (fully braced)



Local buckling check per DSM 1.2.2.2

Per Section 2.2 of this Guide $M_{cr\ell} > 1.66 M_y$, so $\qquad M_{nl} := M_{ne}$

Distortional buckling check per DSM 1.2.2.3

Distortional buckling is not relevant to this cross-section. That may be handled by ignoring M_{nd} or setting M_{nd} to the maximum bending strength, M_y

Predicted flexural strength per DSM 1.3

$M_n := \min((M_{ne}\ \ M_{nl}\ \ M_{nd})) \qquad M_n = 79\ \text{kip} \cdot \text{in}$

The geometry of this section falls within the "pre-qualified" beams of DSM 1.1.1.2 and the higher ϕ and lower Ω of DSM Section 1.2.2 may therefore be used.

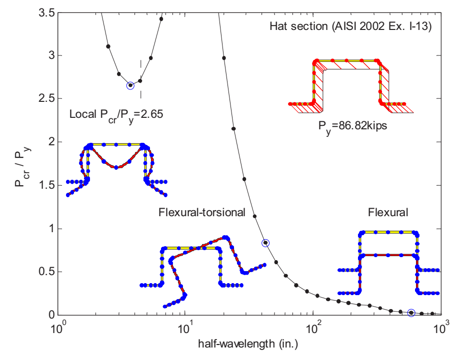
LRFD: $\phi_b := 0.9 \qquad \phi_b \cdot M_n = 71\ \text{kip} \cdot \text{in}$ flexural design strength

ASD: $\Omega_b := 1.67 \qquad \dfrac{M_n}{\Omega_b} = 47\ \text{kip} \cdot \text{in}$ flexural allowable strength

8.9-2 Compressive strength for a continuously braced column

Finite strip analysis of 3HU4.5x135 in pure compression is summarized in Section 3.2.9, and below.

Inputs from the finite strip analysis include:



$P_y := 86.82 \cdot kip$

$P_{crl} := 2.65 \cdot P_y$ $\qquad$ $P_{crl} = 230.1\ kip$

Per DSM 1.2.1, P_n is the minimum of P_{ne}, $P_{n\ell}$, P_{nd}. If a column is continuously braced then global buckling P_{ne} is restricted and the squash load will develop if the section is compact.

$P_{ne} := P_y$ $\qquad$ $P_{ne} = 86.82\ kip$

Local buckling check per DSM 1.2.1.2

Per Section 2.2 of this Guide $P_{cr\ell}$>1.66P_y, so $\qquad$ $P_{nl} := P_{ne}$

Distortional buckling check per DSM 1.2.1.3

In this example, it is presumed that the continuous bracing of the global modes will also restrict any distortional mode that may occur. This may be handled by ignoring P_{nd} or setting P_{nd} to the maximum compressive strength, P_y

Predicted compressive strength per DSM 1.2

$P_n := \min\left(\left(P_{ne}\ \ P_{nl}\ \ P_{nd}\right)\right)$ $\qquad$ $P_n = 86.8\ kip$

Checking the geometric limits of section 1.1.1.1

h_o/t	$\frac{4.5}{.135} = 33.33$	<<50 therefore OK	
b_o/t	$\frac{3.0}{.135} = 22.22$	>>20 therefore NG	
D/t	$\frac{1.67}{.135} = 12.37$	>>6 therefore NG	

The cross-section does not meet the limits for a pre-qualified hat section. By DSM the cross-section is not predicted to experience any reduction due to local buckling, therefore it may seem reasonable to extend the boundaries of the pre-qualified cross-sections in this case. Further, the cross-section fits well within the bounds established for C-shaped columns. Nonetheless without supplemental testing or analysis the section is not pre-qualified.

The geometry of this section does not fall within the "pre-qualified" columns of DSM 1.1.1.1 and the lower ϕ and higher Ω of main *Specification* Section A1.1(b) must therefore be used.

LRFD: $\qquad$ $\phi_c := 0.8$ $\qquad$ $\phi_c \cdot P_n = 69.5\ kip$

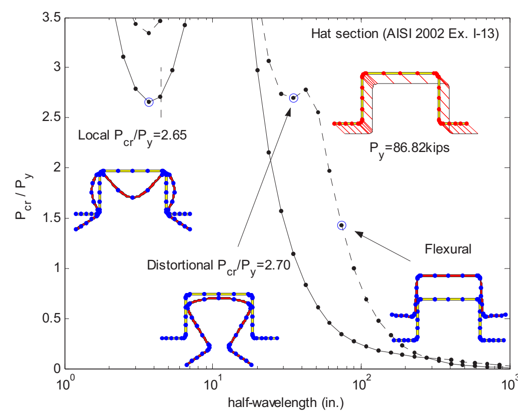
ASD: $\qquad$ $\Omega_c := 2.00$ $\qquad$ $\frac{P_n}{\Omega_c} = 43.41\ kip$

8.9-3 Compressive strength for L=6 ft (AISI 2002 Example III-7)

AISI (2002) Example III-7 examines the beam-column strength of this hat section as a 6 ft long member. This example covers the compressive strength of the column by the Direct Strength Method. It is assumed in the analysis that the hat section is continuously braced against lateral and torsional movement, but the cross-section is free to buckle in the plane perpendicular to the flange.

In the continuously braced column of Example 8.9-2 above, it is assumed that all global buckling modes are restricted. However, in this scenario strong axis flexural buckling is allowed. The hand formulas of the main *Specification* (C4) may be used, but in this example a typical finite strip analysis is examined in finer detail to provide the desired result.

Finite strip analysis results:



The results for the finite strip analysis are shown to the right. At intermediate to long lengths the first mode is torsion, which is restrained, the second mode is flexure. (For further discussion on higher modes, see Chapter 3 of the Guide; specifically Sections 3.3.5, 3.3.7 and 3.3.11). In addition, the distortional mode is also possible if torsion is restricted.

$P_{crl} := 2.65 \cdot P_y$ same as in Example 8.9-3 above

$P_{crd} := 2.70 \cdot P_y$ second mode result

$P_{cre} := 1.51 \cdot P_y$ at 72 in., second mode, flexure

Flexural, torsional, or torsional-flexural check per DSM 1.2.1.1

Here, as described above, only flexure is checked since torsion is assumed restrained.

$$\lambda_c := \sqrt{\frac{P_y}{P_{cre}}} \qquad \lambda_c = 0.81 \qquad \text{(inelastic regime of the column curve)} \qquad \text{(Eq. 1.2.1-1)}$$

$$P_{ne} := \begin{cases} 0.658^{\lambda_c^2} \cdot P_y & \text{if } \lambda_c \le 1.5 \quad \text{(Eq. 1.2.1-2)} \\ \dfrac{0.877}{\lambda_c^2} \cdot P_y & \text{if } \lambda_c > 1.5 \quad \text{(Eq. 1.2.1-3)} \end{cases}$$

$P_{ne} = 65.8\,\text{kip}$

The stress associated with this load is $\frac{P_{ne}}{A_g} = 37.9\,\text{ksi}$ which is F_n in the main *Specification* (compare with 38.4 ksi in Example III-7)

(continued) 8.9-3 Compressive strength for L=6 ft (AISI 2002 Example III-7)

Local buckling check per DSM 1.2.1.2

Per 2.2 of this guide $P_{cr\ell}$>1.66P_y, so $P_{n\ell}$=P_{ne} $\quad$ $P_{nl} := P_{ne}$

Distortional buckling check per DSM 1.2.1.3

$$\lambda_d := \sqrt{\frac{P_y}{P_{crd}}} \qquad \lambda_d = 0.61 \qquad \text{(Eq. 1.2.1-10)}$$

$$P_{nd} := \begin{cases} P_y & \text{if } \lambda_d \le 0.561 \quad \text{(Eq. 1.2.1-8)} \\ \left[1 - 0.25\cdot\left(\frac{P_{crd}}{P_y}\right)^{0.6}\right]\left(\frac{P_{crd}}{P_y}\right)^{0.6}\cdot P_y & \text{if } \lambda_d > 0.561 \quad \text{(Eq. 1.2.1-9)} \end{cases}$$

$P_{nd} = 86.1\,\text{kip}$ (at this length this does not control; however P_{nd} is slightly lower than P_y so at a short enough length P_{nd} will control the strength)

Predicted compressive strength per DSM 1.2

$P_n := \min((P_{ne}\ \ P_{nl}\ \ P_{nd}))$ $\qquad$ $P_n = 65.8\,\text{kip}$

The geometry of this section does not fall within the "pre-qualified" columns of DSM 1.1.1.1 and the lower ϕ and higher Ω of main *Specification* Section A1.1(b) must therefore be used.

LRFD: $\phi_c := 0.8$ $\qquad$ $\phi_c\cdot P_n = 52.6\,\text{kip}$

ASD: $\Omega_c := 2.00$ $\qquad$ $\frac{P_n}{\Omega_c} = 32.9\,\text{kip}$

8.9-4 Beam-column allowable strength (AISI 2002 Example III-7)

AISI (2002) Design Manual Example III-7 examines the capacity of this hat section as a 6 ft. beam-column under uniform load *w* and axial load *P*. Consider the same beam-column as calculated via the Direct Strength Method here following the main *Specification* methodology.

Compression: the compressive strength of this cross-section as determined in Example 8.9-3.

$\Omega_c = 2 \qquad P_n = 65.8\,\text{kip}$

for the interaction equation the fully braced compression strength is needed, per Example 8.9-2.

$P_{no} = 86.8\,\text{kip}$

Bending: as discussed in Example 8.9-3 the cross-section is fully braced against lateral and torsional movement, so the flexural strength is that of Example 8.9-1 above

$\Omega_b = 1.67 \qquad M_n = 79.09\,\text{kip}\cdot\text{in}$

Factors to account for approximate second order analysis

$M := 24.3\cdot\text{kip}\cdot\text{in}$ The first order required allowable strength from AISI 2002 Example III-7

$C_m := 1.0$ The member is pinned at its ends with a uniform load so the max. 2nd order (amplified) moments and the primary moments are at the same location and C_m should be 1.0.

$\alpha := 1 - \dfrac{P}{P_E}$ α is the moment amplification term for weak-axis bending. The required allowable axial strength P is given in Example III-7 as 12.0 kips, the elastic buckling load about the weak axis can be determined by Eq. C5.2.1-7 in the main *Specification*, or taken from the finite strip analysis.

$P := 12\cdot\text{kip}$

$P_E := 138.7\cdot\text{kip}$ from Eq. C5.2.1-7 as used in AISI (2002) Example III-7

or $P_E := P_{cre} \quad P_E = 131.1\,\text{kip}$ from FSM analysis given in Example 8.9-3 above. (This FSM result is used in this example).

$\alpha = 0.91$

Note, the second order required moment is approximated as $\dfrac{C_m\cdot M}{\alpha} = 26.6\,\text{kip}\cdot\text{in}$

Interaction equations

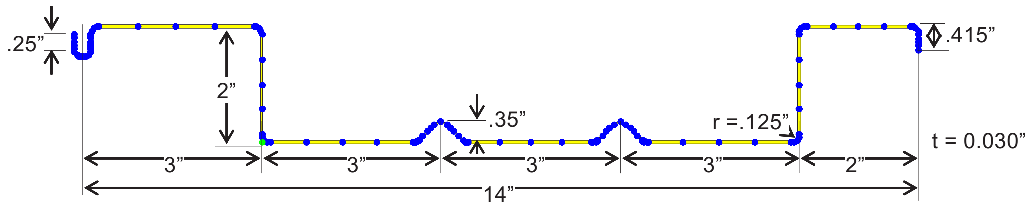
$\dfrac{\Omega_c\cdot P}{P_n} = 0.36$ which is > 0.15, therefore use Equations C5.2.1-1 and C5.2.1-2

$$\frac{\Omega_c\cdot P}{P_n} + \frac{\Omega_b\cdot C_m\cdot M}{M_n\cdot\alpha} = 0.93 \quad \text{OK!} \qquad \text{(Eq. C5.2.1-1)}$$

$$\frac{\Omega_c\cdot P}{P_{no}} + \frac{\Omega_b\cdot M}{M_n} = 0.79 \quad \text{OK!} \qquad \text{(Eq. C5.2.1-2)}$$

for format of LRFD solution see AISI (2002) Design Manual Example III-7.

8.10 Panel section



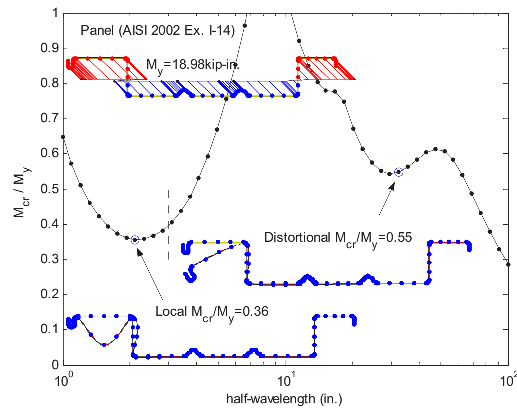
Given:
a. Steel: $F_y = 50$ ksi
b. 14 in. x 2 in. panel as shown above
c. Finite strip analysis results (Section 3.2.10)

Required:
1. Flexural strength for top flange in compression
   a. edges free (as in an end panel)
   b. edges tied (as in a center/repeated panel)
2. Flexural strength for bottom flange in compression

8.10-1 Flexural strength for top flange in compression

a. edges free (as in an end panel)

Finite strip analysis results:



Inputs from the finite strip analysis include:

$M_y := 18.98 \cdot \text{kip} \cdot \text{in}$

$M_{crl} := 0.36 \cdot M_y$ $\quad\quad$ $M_{crl} = 7\,\text{kip} \cdot \text{in}$

$M_{crd} := 0.55 \cdot M_y$ $\quad\quad$ $M_{crd} = 10\,\text{kip} \cdot \text{in}$

Per DSM 1.2.2, M_n is the minimum of M_{ne}, $M_{n\ell}$, M_{nd}. Assuming whole panel lateral-torsional buckling is restricted (braced) then $M_{ne} = M_y$, $M_{n\ell}$ and M_{nd} must still be checked.

$M_{ne} := M_y$ $\quad\quad$ $M_{ne} = 19\,\text{kip} \cdot \text{in}$ $\quad\quad$ (fully braced)

Local buckling check per DSM 1.2.2.2

$$\lambda_l := \sqrt{\frac{M_{ne}}{M_{crl}}} \quad\quad \lambda_l = 1.67 \quad\quad (\text{subscript "l" = "}\ell\text{"}) \quad\quad \text{(Eq. 1.2.2-7)}$$

$$M_{nl} := \begin{cases} M_{ne} & \text{if } \lambda_l \le 0.776 \\ \left[1 - 0.15 \cdot \left(\frac{M_{crl}}{M_{ne}}\right)^{0.4}\right]\left(\frac{M_{crl}}{M_{ne}}\right)^{0.4} \cdot M_{ne} & \text{if } \lambda_l > 0.776 \end{cases} \quad\quad \text{(Eq. 1.2.2-5)} \; \text{(Eq. 1.2.2-6)}$$

$M_{nl} = 11.36\,\text{kip} \cdot \text{in}$

(continued) 8.10-1 Flexural strength for top flange in compression

Distortional buckling check per DSM 1.2.2.3

$$\lambda_d := \sqrt{\frac{M_y}{M_{crd}}} \qquad \lambda_d = 1.35 \qquad \text{(Eq. 1.2.2-10)}$$

$$M_{nd} := \begin{cases} M_y & \text{if } \lambda_d \le 0.673 \\ \left[1 - 0.22 \cdot \left(\frac{M_{crd}}{M_y}\right)^{0.5}\right]\left(\frac{M_{crd}}{M_y}\right)^{0.5} \cdot M_y & \text{if } \lambda_d > 0.673 \end{cases} \qquad \begin{matrix}\text{(Eq. 1.2.2-8)} \\ \text{(Eq. 1.2.2-9)}\end{matrix}$$

$$M_{nd} = 11.78\,\text{kip}\cdot\text{in}$$

Predicted flexural strength per DSM 1.3

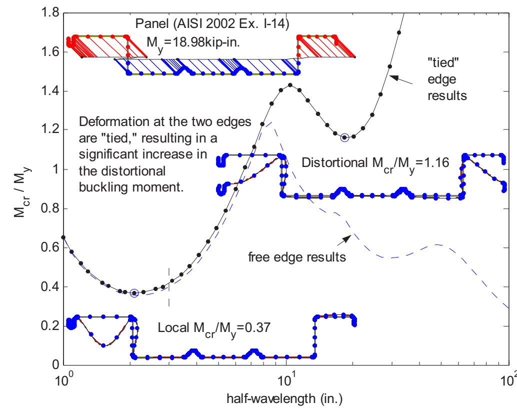
$$M_n := \min\left(\left(M_{ne} \;\; M_{nl} \;\; M_{nd}\right)\right) \qquad M_n = 11.4\,\text{kip}\cdot\text{in}$$

The geometry of this section falls outside the "pre-qualified" beams of DSM 1.1.1.2 and the lower ϕ and higher Ω of main *Specification* Section A1.1(b) are therefore used.

LRFD: $\phi_b := 0.8 \qquad \phi_b \cdot M_n = 9.1\,\text{kip}\cdot\text{in}$

ASD: $\Omega_b := 2.0 \qquad \dfrac{M_n}{\Omega_b} = 5.7\,\text{kip}\cdot\text{in}$

(continued) 8.10-1 Flexural strength for top flange in compression



b. edges tied (as in a center/repeated panel)

Finite strip analysis results:

Inputs from the finite strip analysis include:

$M_y := 18.98 \cdot kip \cdot in$

$M_{crl} := 0.37 \cdot M_y$ $\qquad M_{crl} = 7\,kip \cdot in$

$M_{crd} := 1.16 \cdot M_y$ $\qquad M_{crd} = 22\,kip \cdot in$

Per DSM 1.2.2, M_n is the minimum of M_{ne}, $M_{n\ell}$, M_{nd}. Assuming whole panel lateral-torsional buckling is restricted (braced) then $M_{ne} = M_y$, $M_{n\ell}$ and M_{nd} must still be checked.

$M_{ne} := M_y$ $\qquad M_{ne} = 19\,kip \cdot in$ $\qquad$ (fully braced)

Local buckling check per DSM 1.2.2.2

$$\lambda_l := \sqrt{\frac{M_{ne}}{M_{crl}}} \qquad \lambda_l = 1.64 \qquad (\text{subscript "l" = "}\ell\text{"}) \qquad (\text{Eq. 1.2.2-7})$$

$$M_{nl} := \begin{cases} M_{ne} & \text{if } \lambda_l \le 0.776 \\ \left[1 - 0.15 \cdot \left(\frac{M_{crl}}{M_{ne}}\right)^{0.4}\right]\left(\frac{M_{crl}}{M_{ne}}\right)^{0.4} \cdot M_{ne} & \text{if } \lambda_l > 0.776 \end{cases} \qquad \begin{matrix}(\text{Eq. 1.2.2-5})\\(\text{Eq. 1.2.2-6})\end{matrix}$$

$M_{nl} = 11\,kip \cdot in$

Distortional buckling check per DSM 1.2.2.3

$$\lambda_d := \sqrt{\frac{M_y}{M_{crd}}} \qquad \lambda_d = 0.93 \qquad (\text{Eq. 1.2.2-10})$$

$$M_{nd} := \begin{cases} M_y & \text{if } \lambda_d \le 0.673 \\ \left[1 - 0.22 \cdot \left(\frac{M_{crd}}{M_y}\right)^{0.5}\right]\left(\frac{M_{crd}}{M_y}\right)^{0.5} \cdot M_y & \text{if } \lambda_d > 0.673 \end{cases} \qquad \begin{matrix}(\text{Eq. 1.2.2-8})\\(\text{Eq. 1.2.2-9})\end{matrix}$$

$M_{nd} = 16\,kip \cdot in$ (up from 12 kip-in. when the boundary condition of the edge is left free)

Predicted flexural strength per DSM 1.3

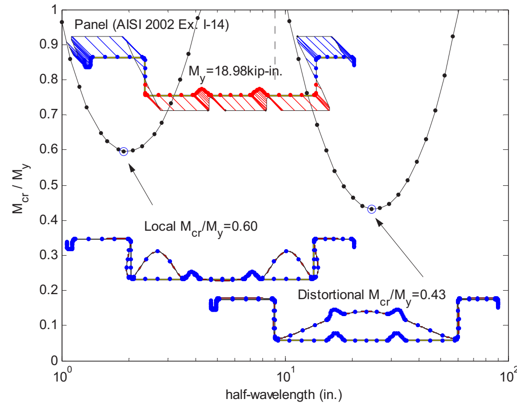
$M_n := \min((M_{ne}\ \ M_{nl}\ \ M_{nd}))$ $\qquad M_n = 11.5\,kip \cdot in$

The geometry of this section falls outside the "pre-qualified" beams of DSM 1.1.1.2 and the lower ϕ and higher Ω of main *Specification* Section A1.1(b) are therefore used.

LRFD: $\quad \phi_b := 0.8 \quad \phi_b \cdot M_n = 9.2\,kip \cdot in$ $\qquad$ ASD: $\quad \Omega_b := 2.0 \quad \frac{M_n}{\Omega_b} = 6\,kip \cdot in$

8.10-2 Flexural strength for bottom flange in compression

Finite strip analysis results:



Inputs from the finite strip analysis include:

$M_y := 18.98 \cdot kip \cdot in$

$M_{crl} := 0.60 \cdot M_y$ $\qquad$ $M_{crl} = 11\ kip \cdot in$

$M_{crd} := 0.43 \cdot M_y$ $\qquad$ $M_{crd} = 8\ kip \cdot in$

Per DSM 1.2.2, M_n is the minimum of M_{ne}, $M_{n\ell}$, M_{nd}. Assuming whole panel lateral-torsional buckling is restricted (braced) then $M_{ne} = M_y$, $M_{n\ell}$ and M_{nd} must still be checked.

$M_{ne} := M_y$ $\qquad$ $M_{ne} = 19\ kip \cdot in$ $\qquad$ (fully braced)

Local buckling check per DSM 1.2.2.2

$$\lambda_l := \sqrt{\frac{M_{ne}}{M_{crl}}} \qquad \lambda_l = 1.29 \qquad \text{(subscript "l" = "}\ell\text{")} \qquad \text{(Eq. 1.2.2-7)}$$

$$M_{nl} := \begin{cases} M_{ne} & \text{if } \lambda_l \le 0.776 \\ \left[1 - 0.15 \cdot \left(\frac{M_{crl}}{M_{ne}}\right)^{0.4}\right] \left(\frac{M_{crl}}{M_{ne}}\right)^{0.4} \cdot M_{ne} & \text{if } \lambda_l > 0.776 \end{cases} \qquad \text{(Eq. 1.2.2-5)} \ \text{(Eq. 1.2.2-6)}$$

$M_{nl} = 14\ kip \cdot in$

Distortional buckling check per DSM 1.2.2.3

$$\lambda_d := \sqrt{\frac{M_y}{M_{crd}}} \qquad \lambda_d = 1.52 \qquad \text{(Eq. 1.2.2-10)}$$

$$M_{nd} := \begin{cases} M_y & \text{if } \lambda_d \le 0.673 \\ \left[1 - 0.22 \cdot \left(\frac{M_{crd}}{M_y}\right)^{0.5}\right] \left(\frac{M_{crd}}{M_y}\right)^{0.5} \cdot M_y & \text{if } \lambda_d > 0.673 \end{cases} \qquad \text{(Eq. 1.2.2-8)} \ \text{(Eq. 1.2.2-9)}$$

$M_{nd} = 11\ kip \cdot in$

Predicted flexural strength per DSM 1.3

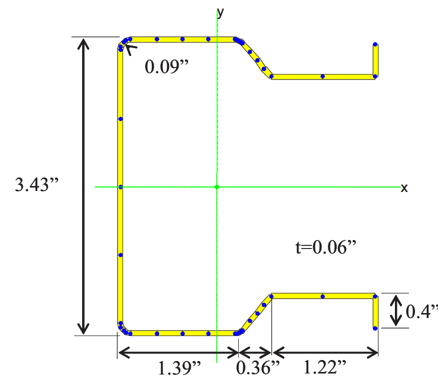
$M_n := \min((M_{ne}\ \ M_{nl}\ \ M_{nd}))$ $\qquad$ $M_n = 10.7\ kip \cdot in$

The geometry of this section falls outside the "pre-qualified" beams of DSM 1.1.1.2 and the lower ϕ and higher Ω of main *Specification* Section A1.1(b) are therefore used.

LRFD: $\phi_b := 0.8$ $\quad$ $\phi_b \cdot M_n = 8.5\ kip \cdot in$ $\qquad$ ASD: $\Omega_b := 2.0$ $\quad$ $\frac{M_n}{\Omega_b} = 5.3\ kip \cdot in$

8.11 Rack post section

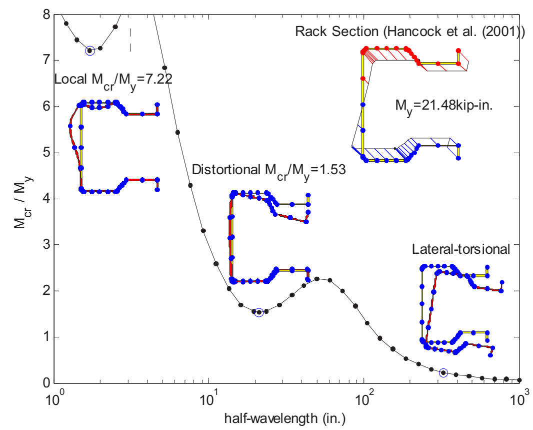
Given:
a. Steel: $F_y = 33$ ksi
b. Section as shown to the right
c. Finite strip analysis results (Section 3.2.11)
Required:
1. Flexural strength about x-axis for a fully braced member
2. Flexural strength about y-axis for a fully braced member
3. Compressive strength for a continuously braced column



8.11-1 Flexural strength about x-axis for a fully braced member

Determination of the bending strength for a fully braced member is equivalent to determining the effective section modulus at yield in the main *Specification*.

Finite strip analysis of rack in pure bending:



Inputs from the finite strip analysis include:

$M_y := 21.48 \cdot \text{kip} \cdot \text{in}$

$M_{crl} := 7.22 \cdot M_y$ $\quad\quad M_{crl} = 155 \, \text{kip} \cdot \text{in}$

$M_{crd} := 1.53 \cdot M_y$ $\quad\quad M_{crd} = 33 \, \text{kip} \cdot \text{in}$

Per DSM 1.2.2, M_n is the minimum of M_{ne}, $M_{n\ell}$, M_{nd}. For a fully braced member lateral-torsional buckling will not occur and thus $M_{ne} = M_y$, $M_{n\ell}$ and M_{nd} must still be checked.

$M_{ne} := M_y$ $\quad\quad M_{ne} = 21 \, \text{kip} \cdot \text{in}$ $\quad\quad$ (fully braced)

Local buckling check per DSM 1.2.2.2

$\lambda_\ell < 0.776$ therefore $\quad M_{nl} := M_{ne}$ $\quad\quad$ (Eq. 1.2.2-5)

Distortional buckling check per DSM 1.2.2.3

$$\lambda_d := \sqrt{\frac{M_y}{M_{crd}}} \quad\quad \lambda_d = 0.81 \quad\quad \text{(Eq. 1.2.2-10)}$$

$$M_{nd} := \begin{cases} M_y & \text{if } \lambda_d \le 0.673 \\ \left[1 - 0.22 \cdot \left(\frac{M_{crd}}{M_y}\right)^{0.5}\right]\left(\frac{M_{crd}}{M_y}\right)^{0.5} \cdot M_y & \text{if } \lambda_d > 0.673 \end{cases} \quad\quad \begin{matrix}\text{(Eq. 1.2.2-8)} \\ \text{(Eq. 1.2.2-9)}\end{matrix}$$

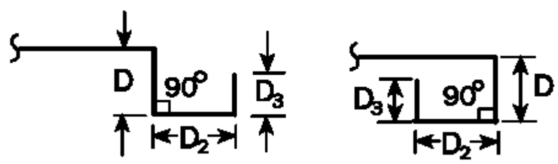
$M_{nd} = 19 \, \text{kip} \cdot \text{in}$

(Continued) 8.11-1 Flexural strength about x-axis for a fully braced member

Predicted flexural strength per DSM 1.3

$M_n := \min((M_{ne}\ \ M_{nl}\ \ M_{nd}))$ $\quad M_n = 19\,\text{kip}\cdot\text{in}$

For determining ϕ the DSM as published in the AISI (2004) Supplement does not include any pre-qualified rack sections in bending. However, since publication of the Supplement AISI has passed a ballot that extends the coverage for C-sections to those sections which have complex stiffeners, similar to those used in rack post uprights:



For complex lips:
- $D_2/t < 17$
- $D_2/D < 0.35$
- $D_3/t < 6$
- $D_2/D3 < 1$

The rack section of this example meets the criteria for a standard C-section, but the complex stiffening lip must still be checked.

D_2/t $\quad \frac{1.22}{0.06} = 20.33$ $\quad < 17?$ no - NG

D_2/D $\quad \frac{1.22}{0.594} = 2.05$ $\quad < 0.35?$ no- NG

D_3/t $\quad \frac{0.4}{0.06} = 6.67$ $\quad < 6?$ no - NG

D_2/D_3 $\quad \frac{1.22}{0.4} = 3.05$ $\quad < 1?$ no -NG

The analyzed section fails the test for pre-qualified beams.This specimen should use the rational analysis ϕ and Ω values from Section A1.1(b) of the main *Specification*.

Note, the angle of the first return lip may be +/- 90 degrees as shown in the figures above.

The section may not be considered pre-qualified.

LRFD: $\quad \phi_b := 0.8 \quad \phi_b \cdot M_n = 15\,\text{kip}\cdot\text{in}$

ASD: $\quad \Omega_b := 2.00 \quad \frac{M_n}{\Omega_b} = 10\,\text{kip}\cdot\text{in}$

8.11-2 Flexural strength about y-axis for a fully braced member

Determination of the bending strength for a fully braced member is equivalent to determining the effective section modulus at yield in the main *Specification*.

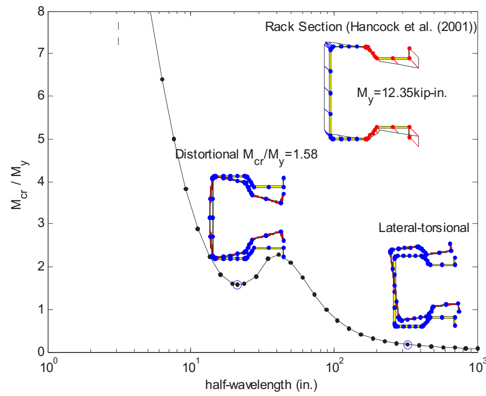
Finite strip analysis in pure bending about y-axis:

Inputs from the finite strip analysis include:

$M_y := 12.35 \cdot kip \cdot in$

$M_{crl} > 8\ M_y$

$M_{crd} := 1.58 \cdot M_y$ $\qquad M_{crd} = 20\,kip \cdot in$



Per DSM 1.2.2, M_n is the minimum of M_{ne}, $M_{n\ell}$, M_{nd}. For a fully braced member lateral-torsional buckling will not occur and thus $M_{ne} = M_y$, $M_{n\ell}$ and M_{nd} must still be checked.

$M_{ne} := M_y$ $\qquad M_{ne} = 12\,kip \cdot in$ $\qquad$ (fully braced)

Local buckling check per DSM 1.2.2.2

$\lambda_\ell < 0.776$ therefore $\quad M_{nl} := M_{ne}$ $\qquad$ (Eq. 1.2.2-5)

Distortional buckling check per DSM 1.2.2.3

$$\lambda_d := \sqrt{\frac{M_y}{M_{crd}}} \qquad \lambda_d = 0.8 \qquad \text{(Eq. 1.2.2-10)}$$

$$M_{nd} := \begin{cases} M_y & \text{if } \lambda_d \le 0.673 \\ \left[1 - 0.22 \cdot \left(\frac{M_{crd}}{M_y}\right)^{0.5}\right]\left(\frac{M_{crd}}{M_y}\right)^{0.5} \cdot M_y & \text{if } \lambda_d > 0.673 \end{cases} \qquad \begin{matrix}\text{(Eq. 1.2.2-8)} \\ \text{(Eq. 1.2.2-9)}\end{matrix}$$

$M_{nd} = 11\,kip \cdot in$

Predicted flexural strength per DSM 1.3

$M_n := \min((M_{ne}\ \ M_{nl}\ \ M_{nd}))$ $\qquad M_n = 11\,kip \cdot in$

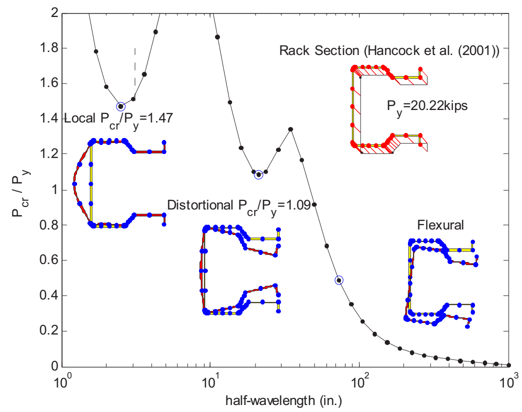
The geometry of this section does not fall within the "pre-qualified" beams of DSM 1.1.1.2. and the lower ϕ and higher Ω of main *Specification* Section A1.1(b) must therefore be used.

LRFD: $\quad \phi_b := 0.8 \qquad \phi_b \cdot M_n = 8.98\,kip \cdot in$

ASD: $\quad \Omega_b := 2.0 \qquad \frac{M_n}{\Omega_b} = 5.62\,kip \cdot in$

8.11-3 Compressive strength for a continuously braced column

Finite strip analysis in pure compression:



Inputs from the finite strip analysis include:

$P_y := 20.22 \cdot kip$

$P_{crl} := 1.47 \cdot P_y$ $\quad$ $P_{crl} = 29.7\,kip$

$P_{crd} := 1.09 \cdot P_y$ $\quad$ $P_{crd} = 22\,kip$

If a column is continuously braced then global buckling P_{ne} is restricted and the long column strength is simply the squash load.

$P_{ne} := P_y$ $\quad$ $P_{ne} = 20.22\,kip$

Local buckling check per DSM 1.2.1.2

$$\lambda_l := \sqrt{\frac{P_{ne}}{P_{crl}}} \qquad \lambda_l = 0.82 \qquad \text{(subscript "l" = "}\ell\text{")} \qquad \text{(Eq. 1.2.1-7)}$$

$$P_{nl} := \begin{cases} P_{ne} & \text{if } \lambda_l \le 0.776 \\ \left[1 - 0.15 \cdot \left(\frac{P_{crl}}{P_{ne}}\right)^{0.4}\right]\left(\frac{P_{crl}}{P_{ne}}\right)^{0.4} \cdot P_{ne} & \text{if } \lambda_l > 0.776 \end{cases} \qquad \begin{matrix}\text{(Eq. 1.2.1-5)}\\ \text{(Eq. 1.2.1-6)}\end{matrix}$$

$P_{nl} = 19.46\,kip$

Distortional buckling check per DSM 1.2.1.3

$$\lambda_d := \sqrt{\frac{P_y}{P_{crd}}} \qquad \lambda_d = 0.96 \qquad \text{(Eq. 1.2.1-10)}$$

$$P_{nd} := \begin{cases} P_y & \text{if } \lambda_d \le 0.561 \\ \left[1 - 0.25 \cdot \left(\frac{P_{crd}}{P_y}\right)^{0.6}\right]\left(\frac{P_{crd}}{P_y}\right)^{0.6} \cdot P_y & \text{if } \lambda_d > 0.561 \end{cases} \qquad \begin{matrix}\text{(Eq. 1.2.1-8)}\\ \text{(Eq. 1.2.1-9)}\end{matrix}$$

$P_{nd} = 15.7\,kip$

(Continued) 8.11-3 Compressive strength for a continuously braced column

Predicted compressive strength per DSM 1.2

$P_n := \min((P_{ne} \;\; P_{nl} \;\; P_{nd}))$ $\qquad P_n = 15.7\,\text{kip}$

For determining ϕ, check against the pre-qualified columns of Table 1.1.1-1 in DSM Section 1.1.1.1. In the pre-qualified rack uprights, the first flange lip, D, is always 90 degrees from the flange, and in this example D is 45 degrees. Here we assume this minor change is not a violation of the pre-qualified columns since θ variation was investigated significantly in the reported lipped C-sections, and thus check against the rack uprights given in the table.

$h_o/t \quad \frac{3.43}{0.06} = 57.17 \quad > 51$, therefore NG

$b_o/t \quad \frac{1.39}{0.06} = 23.17 \quad > 22$, therefore NG

$D/t \quad \frac{0.594}{0.06} = 9.9 \quad > 8$, therefore NG

$h_o/b_o \quad \frac{3.43}{1.39} = 2.47 \quad < 2.9$, OK

$b_2/D \quad \frac{1.22}{0.594} = 2.05 \quad > 2$, therefore NG

$D_2/D \quad \frac{0.4}{0.594} = 0.67 \quad > 0.3$, therefore, NG

The analyzed section fails the test for pre-qualified columns. The small violations for h_o/t, b_o/t, and D/t may be deemed legitimate by engineering judgment since the parameters are well within the values tested in standard C-shaped specimens. However, since the details of the complex rack post stiffener also fall outside the bounds of the tested specimens it becomes difficult to argue that this specimen should use the pre-qualified ϕ and Ω values, instead the rational analysis values are appropriate.

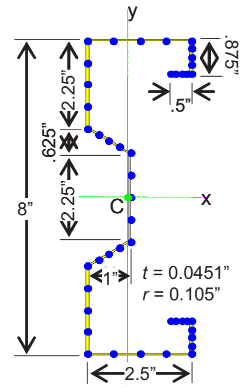
The geometry of this section falls outside the "pre-qualified" columns of DSM 1.1.1.1 and thus the lower ϕ and higher Ω of main *Specification* Section A1.1(b) must be used.

LRFD: $\phi_c := 0.8 \qquad \phi_c \cdot P_n = 12.5\,\text{kip}$

ASD: $\Omega_c := 2.0 \qquad \frac{P_n}{\Omega_c} = 7.8\,\text{kip}$

8.12 Sigma Section

Given:
a. Steel: $F_y = 50$ ksi
b. Section 800SG250-43 as shown to the right
c. Finite strip analysis results (Section 3.2.12)
Required:
1. Flexural strength for a fully braced member
2. Compressive strength for a continuously braced column



(centerline model)

8.12-1 Flexural strength for a fully braced member

Determination of the bending strength for a <u>fully braced member</u> is equivalent to determining the effective section modulus at yield in the main *Specification*.

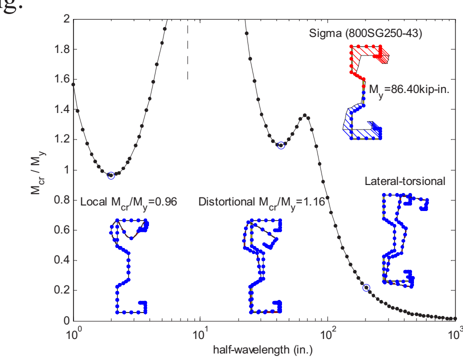
Finite strip analysis of 800SG250-43 in pure bending:

Inputs from the finite strip analysis include:

$M_y := 86.40 \cdot \text{kip} \cdot \text{in}$

$M_{crl} := 0.96 \cdot M_y$ $\qquad M_{crl} = 83\,\text{kip} \cdot \text{in}$

$M_{crd} := 1.16 \cdot M_y$ $\qquad M_{crd} = 100\,\text{kip} \cdot \text{in}$



Per DSM 1.2.2, M_n is the minimum of M_{ne}, $M_{n\ell}$, M_{nd}. For a fully braced member lateral-torsional buckling will not occur and thus $M_{ne} = M_y$, $M_{n\ell}$ and M_{nd} must still be checked.

$M_{ne} := M_y$ $\qquad M_{ne} = 86\,\text{kip} \cdot \text{in}$ $\qquad$ (fully braced)

Local buckling check per DSM 1.2.2.2

$$\lambda_l := \sqrt{\frac{M_{ne}}{M_{crl}}} \qquad \lambda_l = 1.02 \qquad \text{(subscript "l" = "}\ell\text{")} \qquad \text{(Eq. 1.2.2-7)}$$

$$M_{nl} := \begin{cases} M_{ne} & \text{if } \lambda_l \le 0.776 \\ \left[1 - 0.15 \cdot \left(\frac{M_{crl}}{M_{ne}}\right)^{0.4}\right]\left(\frac{M_{crl}}{M_{ne}}\right)^{0.4} \cdot M_{ne} & \text{if } \lambda_l > 0.776 \end{cases} \qquad \text{(Eq. 1.2.2-5)} \; \text{(Eq. 1.2.2-6)}$$

$M_{nl} = 72\,\text{kip} \cdot \text{in}$

(Continued) 8.12-1 Flexural strength for a fully braced member

Distortional buckling check per DSM 1.2.2.3

$\lambda_d := \sqrt{\frac{M_y}{M_{crd}}}$ $\qquad \lambda_d = 0.93$ (Eq. 1.2.2-10)

$$M_{nd} := \begin{cases} M_y & \text{if } \lambda_d \le 0.673 \\ \left[1 - 0.22 \cdot \left(\frac{M_{crd}}{M_y}\right)^{0.5}\right] \left(\frac{M_{crd}}{M_y}\right)^{0.5} \cdot M_y & \text{if } \lambda_d > 0.673 \end{cases}$$

(Eq. 1.2.2-8)
(Eq. 1.2.2-9)

$M_{nd} = 71\,\text{kip}\cdot\text{in}$

Predicted flexural strength per DSM 1.3

$M_n := \min((M_{ne} \ M_{nl} \ M_{nd}))$ $\qquad M_n = 71\,\text{kip}\cdot\text{in}$

The geometry of this section falls outside the "pre-qualified" beams of DSM 1.1.1.2 and the lower ϕ and higher Ω of main *Specification* Section A1.1(b) should be used.

LRFD: $\phi_b := 0.8$ $\qquad \phi_b \cdot M_n = 57\,\text{kip}\cdot\text{in}$

ASD: $\Omega_b := 2.0$ $\qquad \frac{M_n}{\Omega_b} = 36\,\text{kip}\cdot\text{in}$

8.12-2 Compressive strength for a continuously braced column

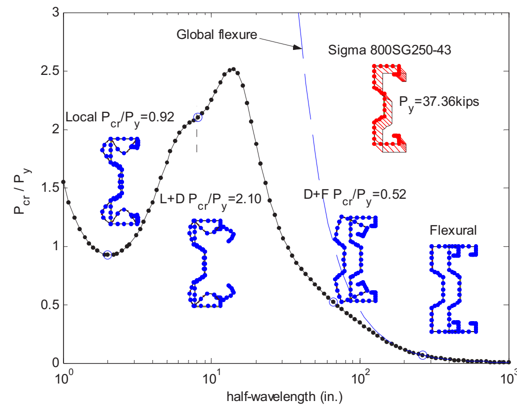
Finite strip analysis of 800SG250-43 in pure compression:

Inputs from the finite strip analysis include:

$P_y := 37.36 \cdot kip$

$P_{crl} := 0.92 \cdot P_y$ $\qquad P_{crl} = 34.4\ kip$

P_{crd} is length dependent



Case I: Continuously braced, bracing restricts distortional buckling

Per DSM 1.2.1, P_n is the minimum of P_{ne}, $P_{n\ell}$, P_{nd}. If a column is continuously braced then global buckling P_{ne} is restricted and the squash load will develop. For this cross-section distortional buckling is also length dependent, in case I, assume that the bracing for global buckling also restricts distortional buckling.

$P_{nd} := P_y$ $\qquad P_{nd} = 37.36\ kip$ $\qquad$ assumed due to bracing

$P_{ne} := P_y$ $\qquad P_{ne} = 37.36\ kip$ $\qquad$ assumed due to bracing

Local buckling check per DSM 1.2.1.2

$$\lambda_l := \sqrt{\frac{P_{ne}}{P_{crl}}} \qquad \lambda_l = 1.04$$

(subscript "l" = "ℓ") (Eq. 1.2.1-7)

$$P_{nl} := \begin{cases} P_{ne} & \text{if } \lambda_l \le 0.776 \\ \left[1 - 0.15 \cdot \left(\frac{P_{crl}}{P_{ne}}\right)^{0.4}\right]\left(\frac{P_{crl}}{P_{ne}}\right)^{0.4} \cdot P_{ne} & \text{if } \lambda_l > 0.776 \end{cases}$$

(Eq. 1.2.1-5)
(Eq. 1.2.1-6)

$P_{nl} = 30.89\ kip$

Predicted compressive strength per DSM 1.2

$P_n := \min\left(\left(P_{ne}\ \ P_{nl}\ \ P_{nd}\right)\right)$ $\qquad P_n = 30.9\ kip$

The geometry of this section falls outside the "pre-qualified" columns of DSM 1.1.1.1 thus the lower ϕ and higher Ω of main *Specification* Section A1.1(b) applies.

LRFD: $\quad \phi_c := 0.8 \quad \phi_c \cdot P_n = 24.7\ kip$

ASD: $\quad \Omega_c := 2.0 \quad \frac{P_n}{\Omega_c} = 15.4\ kip$

(continued) 8.12-2 Compressive strength for a continuously braced column

Case II: Discrete braces at 66 in. that restrict distortional and global buckling

Distortional buckling check per DSM 1.2.1.3

For a 66 in. unbraced length P_{crd} from the FSM analysis is $P_{crd} := 0.52 \cdot P_y$, $P_{crd} = 19.4\,\text{kip}$

$$\lambda_d := \sqrt{\frac{P_y}{P_{crd}}} \qquad \lambda_d = 1.39 \qquad \text{(Eq. 1.2.1-10)}$$

$$P_{nd} := \begin{cases} P_y & \text{if } \lambda_d \le 0.561 \\ \left[1 - 0.25 \cdot \left(\frac{P_{crd}}{P_y}\right)^{0.6}\right]\left(\frac{P_{crd}}{P_y}\right)^{0.6} \cdot P_y & \text{if } \lambda_d > 0.561 \end{cases} \qquad \text{(Eq. 1.2.1-8)} \;\; \text{(Eq. 1.2.1-9)}$$

$P_{nd} = 21\,\text{kip}$ compare with $P_{n\ell}$ from the continuously braced case, $P_{nl} = 30.89\,\text{kip}$

At 66 in. unbraced length, Global buckling must also be calculated!

From the main *Specification* equations (see Section 2.6 of this Guide) or from FSM analysis (see Sections 3.3.4 and 3.3.5 of this Guide). Results from FSM analysis,

$P_{cre} := 1.04 \cdot P_y$ $\qquad P_{cre} = 38.85\,\text{kip}$ at 66 in.

Global buckling (long column strength) check per DSM 1.2.1.1

$$\lambda_c := \sqrt{\frac{P_y}{P_{cre}}} \qquad \lambda_c = 0.98 \quad \text{(inelastic regime)} \qquad \text{(Eq. 1.2.1-3)}$$

$$P_{ne} := \begin{cases} 0.658^{\lambda_c^2} \cdot P_y & \text{if } \lambda_c \le 1.5 \quad \text{(Eq. 1.2.1-1)} \\ \frac{0.877}{\lambda_c^2} \cdot P_y & \text{if } \lambda_c > 1.5 \quad \text{(Eq. 1.2.1-2)} \end{cases}$$

$P_{ne} = 24.98\,\text{kip}$

Local buckling check per DSM 1.2.1.2 (this check is dependent on P_{ne})

$$\lambda_l := \sqrt{\frac{P_{ne}}{P_{crl}}} \qquad \lambda_l = 0.85 \qquad \text{(subscript "l" = "}\ell\text{")} \qquad \text{(Eq. 1.2.1-7)}$$

$$P_{nl} := \begin{cases} P_{ne} & \text{if } \lambda_l \le 0.776 \quad \text{(Eq. 1.2.1-5)} \\ \left[1 - 0.15 \cdot \left(\frac{P_{crl}}{P_{ne}}\right)^{0.4}\right]\left(\frac{P_{crl}}{P_{ne}}\right)^{0.4} \cdot P_{ne} & \text{if } \lambda_l > 0.776 \quad \text{(Eq. 1.2.1-6)} \end{cases}$$

$P_{nl} = 23.55\,\text{kip}$ Distortional buckling controls, but for longer unbraced length $P_{n\ell}$ soon controls.

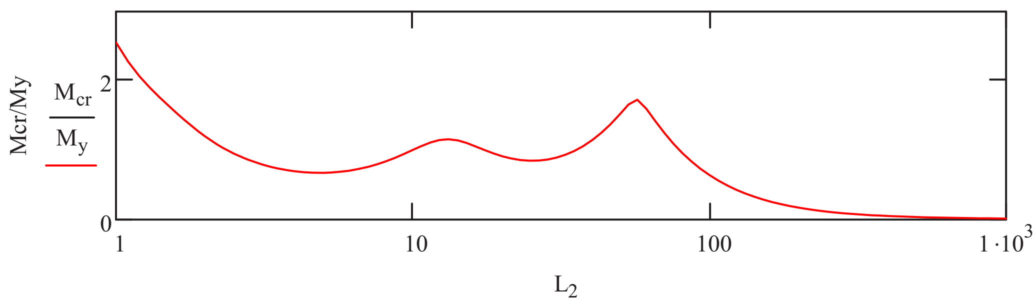
Predicted compressive strength per DSM 1.2

$P_n := \min((P_{ne}\ \ P_{nl}\ \ P_{nd}))$ $\qquad P_n = 21\,\text{kip}$

LRFD: $\phi_c := 0.8$ $\quad \phi_c \cdot P_n = 16.8\,\text{kip}$ $\qquad$ ASD: $\Omega_c := 2.0$ $\quad \frac{P_n}{\Omega_c} = 10.5\,\text{kip}$

8.13 Development of a Beam Chart for the C-section with lips (Section 3.2.1)

Consider again the C-section (9CS2.5x059) in bending with this elastic bucking analysis curve:



half-wavelength (in.)

The finite strip analysis enforces a single half sine wave in developing the above results. To develop a beam chart it is necessary to determine how each of the buckling modes will behave without this restriction. That is to say, find $M_{cr\ell}$, M_{crd}, M_{cre} as a function of length.

Local buckling ($M_{cr\ell}$) as a function of length

For lengths longer than 5 in. (the local buckling minimum in the above figure) local buckling will simply repeat itself undulating up and down. Length shorter than 5 in. is not of interest, so it is assumed that the local buckling value is constant with length.

$M_{crlp} := 0.674 \cdot M_y$ local buckling minimum from FSM

$M_{crl}(L) := M_{crlp}$ local buckling does not change (even at short lengths)

Distortional buckling (M_{crd}) as a function of length

For lengths longer than 25 in. (the distortional buckling minimum in the above figure) distortional buckling will simply repeat with multiple half-waves along the length. For short beams, the increase in distortional buckling as it is restricted may be worthy of investigation. The closed-form solution of Chapter 9 of this Guide could be used, but the simpler empirical expression given below has been found to be adequate.

$M_{crdp} := 0.85 \cdot M_y$ $L_{crd} := 24.8$ distortional buckling minimum from FSM

$$M_{crd}(L) := \begin{cases} M_{crdp} & \text{if } L \ge L_{crd} \\ M_{crdp} \cdot \left(\frac{L}{L_{crd}}\right)^{\ln\left(\frac{L}{L_{crd}}\right)} & \text{if } L < L_{crd} \end{cases}$$

Global buckling (M_{cre}) as a function of length

Global buckling is strongly dependent on length. The closed-form solutions of the main *Specification* (as described in Chapter 9) can be used to provide M_{cre} as a function of length - but here an alternate approach that only employs the FSM analysis is used. This is convenient when quickly exploring the strength of different members and avoids recourse to the longer *Specification* equations and the need for dealing with section properties.

For this case the form of $M_{cre}^2 = \alpha(1/L)^2 + \beta(1/L)^4$ if $L=L_y=L_t$

Pick two points along the global buckling curve to fit to

$L_{cr1} := L2_{70}$ $\quad M_{cre1} := \left(M_{cr_{70}}\right)$ $\quad L_{cr2} := L2_{90}$ $\quad M_{cre2} := \left(M_{cr_{90}}\right)$

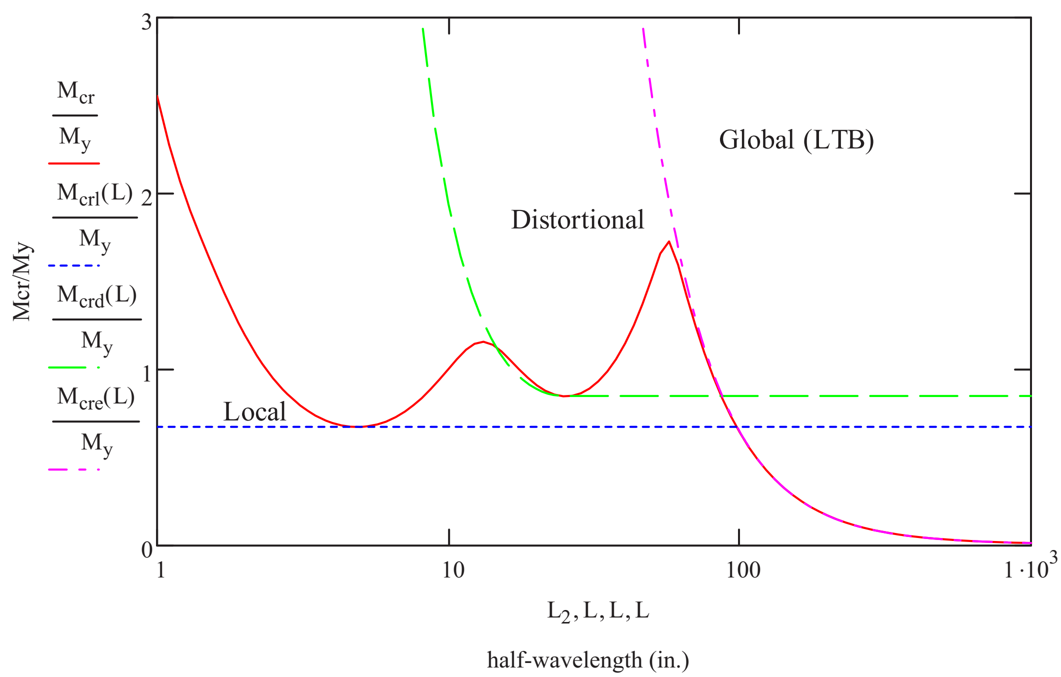
note, here the 70th and 90th points are selected from the FSM analysis. Any two points which are clearly in the global (lateral-torsional) buckling regime will do.

The constants α and β are found (two equations, two unknowns) via:

$$\alpha := \frac{M_{cre1}^2 \cdot L_{cr1}^4 - M_{cre2}^2 \cdot L_{cr2}^4}{L_{cr1}^2 - L_{cr2}^2} \qquad \beta := \frac{\left(M_{cre1}^2 \cdot L_{cr1}^2 - M_{cre2}^2 \cdot L_{cr2}^2\right) \cdot L_{cr1}^2 \cdot L_{cr2}^2}{L_{cr2}^2 - L_{cr1}^2}$$

$$M_{cre}(L) := \sqrt{\alpha \cdot \left(\frac{1}{L}\right)^2 + \beta \cdot \left(\frac{1}{L}\right)^4}$$

based on these assumptions the buckling moments as a function of length are:



Strength predictions, via DSM (Appendix 1 of AISI 2004)

Note, the formulas below are not different from those used extensively in the example problems of this Chapter, except that they are now given as an explicit function of length (L). In this form the equations may be used to directly produce the desired Beam Chart.

Global buckling check per DSM 1.2.2.1

$$M_{ne}(L) := \begin{cases} M_{cre}(L) \ \text{ if } \ M_{cre}(L) < 0.56 \cdot M_y & \text{(Eq. 1.2.2-1)} \\ \frac{10}{9} \cdot M_y \cdot \left(1 - \frac{10 \cdot M_y}{36 \cdot M_{cre}(L)}\right) \ \text{ if } \ 2.78 \cdot M_y \geq M_{cre}(L) \geq 0.56 \cdot M_y & \text{(Eq. 1.2.2-2)} \\ M_y \ \text{ if } \ M_{cre}(L) > 2.78 \cdot M_y & \text{(Eq. 1.2.2-3)} \end{cases}$$

Note: the (L) indicates where the calculations are a function of length

Local buckling check per DSM 1.2.2.2

$$\lambda_l(L) := \sqrt{\frac{M_{ne}(L)}{M_{crl}(L)}} \qquad \text{(subscript "l" = "}\ell\text{")} \qquad \text{(Eq. 1.2.2-7)}$$

$$M_{nl}(L) := \begin{cases} M_{ne}(L) \ \text{ if } \ \lambda_l(L) \leq 0.776 & \text{(Eq. 1.2.2-5)} \\ \left[1 - 0.15 \cdot \left(\frac{M_{crl}(L)}{M_{ne}(L)}\right)^{0.4}\right]\left(\frac{M_{crl}(L)}{M_{ne}(L)}\right)^{0.4} \cdot M_{ne}(L) \ \text{ if } \ \lambda_l(L) > 0.776 & \text{(Eq. 1.2.2-6)} \end{cases}$$

Distortional buckling check per DSM 1.2.2.3

$$\lambda_d(L) := \sqrt{\frac{M_y}{M_{crd}(L)}} \qquad \text{(Eq. 1.2.2-10)}$$

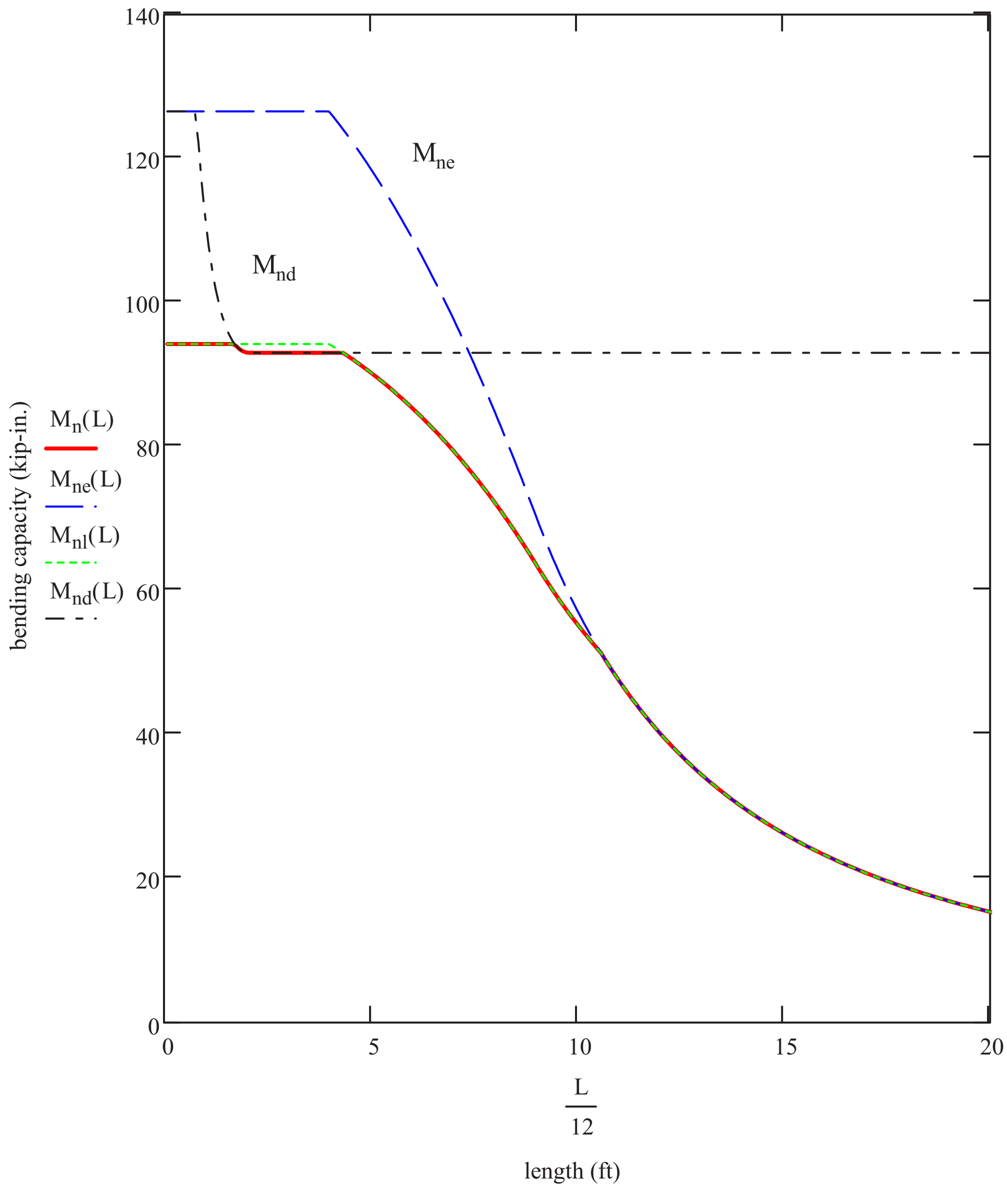
$$M_{nd}(L) := \begin{cases} M_y \ \text{ if } \ \lambda_d(L) \leq 0.673 & \text{(Eq. 1.2.2-8)} \\ \left[1 - 0.22 \cdot \left(\frac{M_{crd}(L)}{M_y}\right)^{0.5}\right]\left(\frac{M_{crd}(L)}{M_y}\right)^{0.5} \cdot M_y \ \text{ if } \ \lambda_d(L) > 0.673 & \text{(Eq. 1.2.2-9)} \end{cases}$$

Predicted flexural strength per DSM 1.3

$$M_n(L) := \min\left(\left(M_{ne}(L) \quad M_{nl}(L) \quad M_{nd}(L)\right)\right)$$

Developed Beam Chart

DSM Beam Chart for the 9CS2.5x059



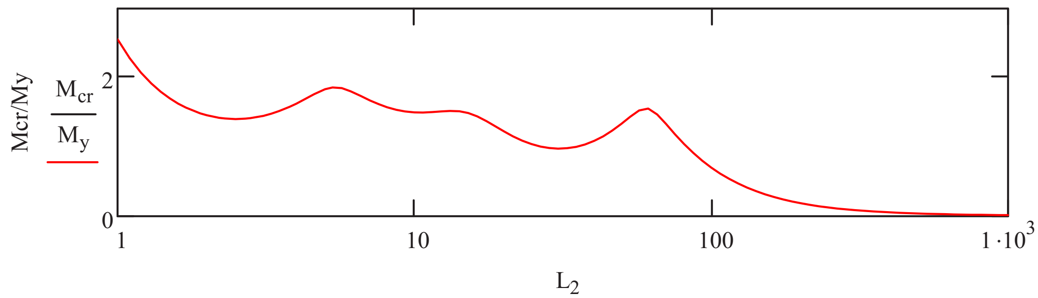
Notes on the beam chart:

(1) Distortional buckling (M_{nd}) controls the strength for a small group of lengths approximately between 2 and 5 ft.

(2) Beyond approximately 10 ft. in length local buckling ($M_{n\ell}$) does not reduce the capacity below the global buckling strength (M_{ne}) (In the parlance of the main *Specification,* this cross-section is fully effective for an unbraced length beyond 10 ft).

Beam chart for the C-section with lips *modified* (Section 3.2.2)

Consider the *modified* C-section with the now familiar elastic bucking analysis curve:



The finite strip analysis enforces a single half sine wave in developing the above results. To develop a beam chart it is necessary to determine how each of the buckling modes will behave without this restriction. That is to say, find $M_{cr\ell}$, M_{crd}, M_{cre} as a function of length.

Local buckling ($M_{cr\ell}$) as a function of length

For lengths longer than 2.5 in. (the local buckling minimum in the above figure) local buckling will simply repeat itself undulating up and down. Length shorter than 2.5 in. is not of interest, so it is assumed that the local buckling value is constant with length.

$M_{crlp} := 1.4 \cdot M_y$ local buckling minimum from FSM

$M_{crl}(L) := M_{crlp}$ local buckling is assumed invariant (even at short lengths)

Distortional buckling (M_{crd}) as a function of length

For lengths longer than 30 in. (the distortional buckling minimum in the above figure) distortional buckling will simply repeat with multiple half-waves along the length. For short beams, the increase in distortional buckling as it is restricted may be worthy of investigation. The closed-form solution of Chapter 9 could be used, but the simpler empirical expression given below has been found to be adequate.

$M_{crdp} := 0.98 \cdot M_y$ $L_{crd} := 30.5$ distortional buckling minimum from FSM

$$M_{crd}(L) := \begin{cases} M_{crdp} & \text{if } L \ge L_{crd} \\ M_{crdp} \cdot \left(\frac{L}{L_{crd}}\right)^{\ln\left(\frac{L}{L_{crd}}\right)} & \text{if } L < L_{crd} \end{cases}$$

Global buckling (M_{cre}) as a function of length

Global buckling is strongly dependent on length. The closed-form solutions of the main *Specification* (as described in Chapter 9) can be used to provide M_{cre} as a function of length - but here an alternate approach that only employs the FSM analysis is used. This is convenient when quickly exploring the strength of different members and avoids recourse to the longer *Specification* equations and the need for dealing with section properties.

For this case the form of $M_{cre}^2 = \alpha(1/L)^2 + \beta(1/L)^4$ if $L=L_y=L_t$

Pick two points along the global buckling curve to fit to

$L_{cr1} := L2_{70}$ $\quad$ $M_{cre1} := \left(M_{cr_{70}}\right)$ $\quad$ $L_{cr2} := L2_{90}$ $\quad$ $M_{cre2} := \left(M_{cr_{90}}\right)$

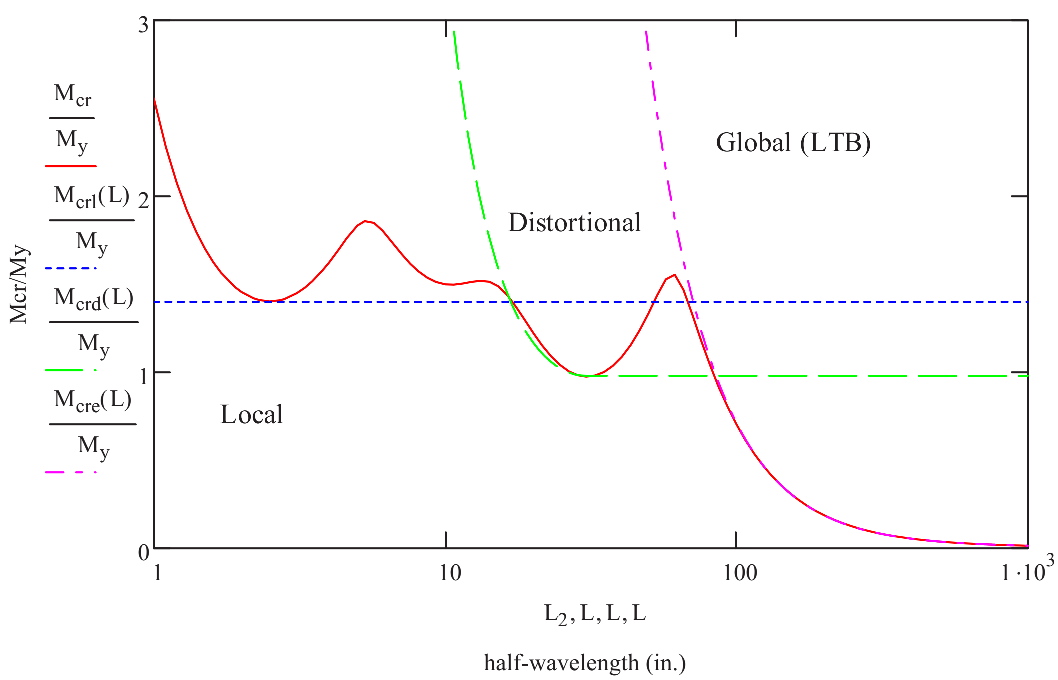
note, here the 70th and 90th points are selected from the FSM analysis. Any two points which are clearly in the global (lateral-torsional) buckling regime will do.

The constants α and β are found (two equations, two unknowns) via:

$$\alpha := \frac{M_{cre1}^{\,2} \cdot L_{cr1}^{\,4} - M_{cre2}^{\,2} \cdot L_{cr2}^{\,4}}{L_{cr1}^{\,2} - L_{cr2}^{\,2}} \qquad \beta := \frac{\left(M_{cre1}^{\,2} \cdot L_{cr1}^{\,2} - M_{cre2}^{\,2} \cdot L_{cr2}^{\,2}\right) \cdot L_{cr1}^{\,2} \cdot L_{cr2}^{\,2}}{L_{cr2}^{\,2} - L_{cr1}^{\,2}}$$

$$M_{cre}(L) := \sqrt{\alpha \cdot \left(\frac{1}{L}\right)^2 + \beta \cdot \left(\frac{1}{L}\right)^4}$$

based on these assumptions the buckling moments as a function of length are:



Strength predictions, via DSM (Appendix 1 of AISI 2004)

Global buckling check per DSM 1.2.2.1

$$M_{ne2}(L) := \begin{cases} M_{cre}(L) \quad \text{if } M_{cre}(L) < 0.56 \cdot M_y & \text{(Eq. 1.2.2-1)} \\ \frac{10}{9} \cdot M_y \cdot \left(1 - \frac{10 \cdot M_y}{36 \cdot M_{cre}(L)}\right) \quad \text{if } 2.78 \cdot M_y \geq M_{cre}(L) \geq 0.56 \cdot M_y & \text{(Eq. 1.2.2-2)} \\ M_y \quad \text{if } M_{cre}(L) > 2.78 \cdot M_y & \text{(Eq. 1.2.2-3)} \end{cases}$$

Local buckling check per DSM 1.2.2.2

$$\lambda_l(L) := \sqrt{\frac{M_{ne}(L)}{M_{crl}(L)}} \qquad \text{(subscript "l" = "}\ell\text{")} \qquad \text{(Eq. 1.2.2-7)}$$

$$M_{nl2}(L) := \begin{cases} M_{ne2}(L) \quad \text{if } \lambda_l(L) \leq 0.776 & \text{(Eq. 1.2.2-5)} \\ \left[1 - 0.15 \cdot \left(\frac{M_{crl}(L)}{M_{ne2}(L)}\right)^{0.4}\right]\left(\frac{M_{crl}(L)}{M_{ne2}(L)}\right)^{0.4} \cdot M_{ne2}(L) \quad \text{if } \lambda_l(L) > 0.776 & \text{(Eq. 1.2.2-6)} \end{cases}$$

Distortional buckling check per DSM 1.2.2.3

$$\lambda_d(L) := \sqrt{\frac{M_y}{M_{crd}(L)}} \qquad \text{(Eq. 1.2.2-10)}$$

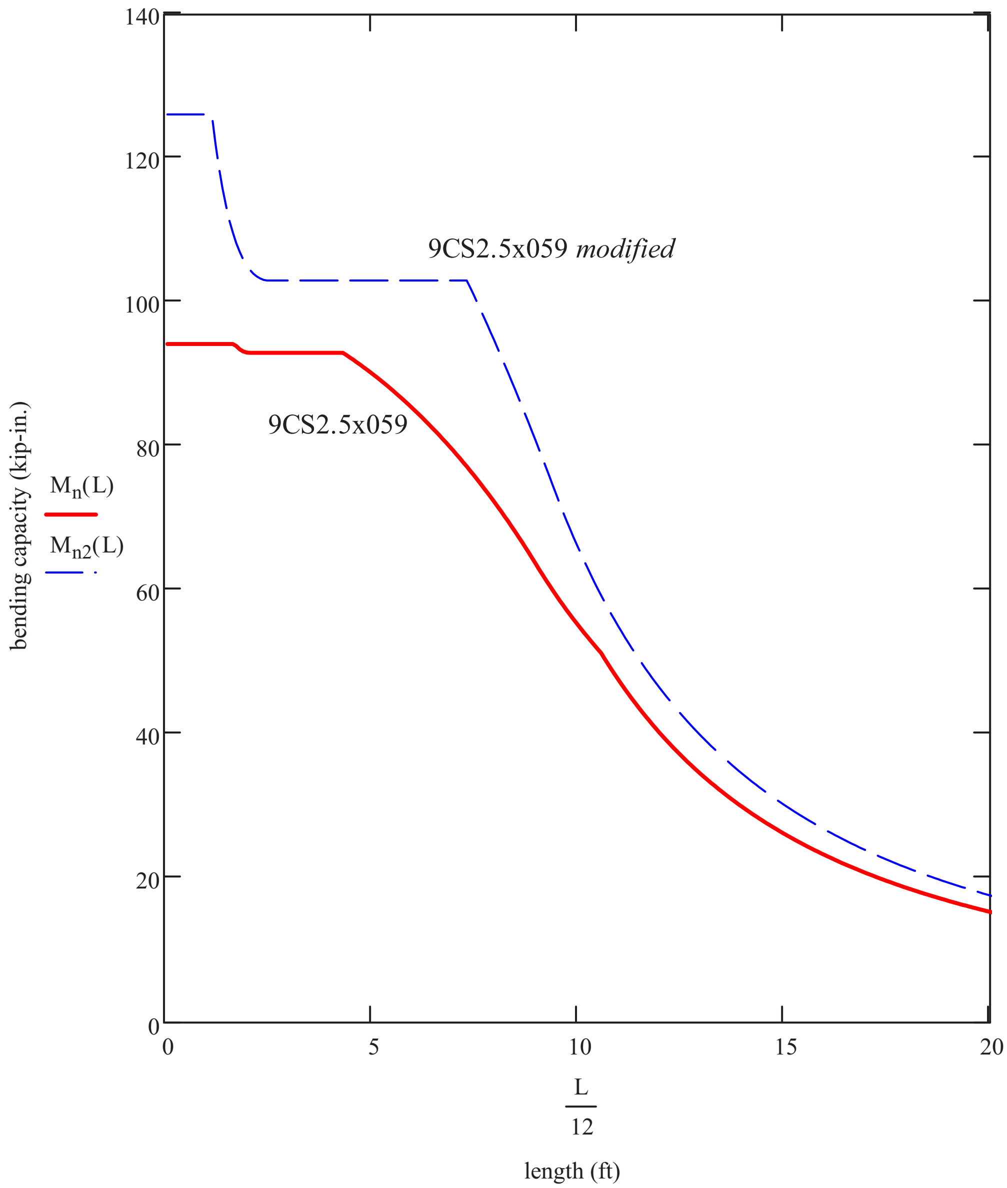
$$M_{nd2}(L) := \begin{cases} M_y \quad \text{if } \lambda_d(L) \leq 0.673 & \text{(Eq. 1.2.2-8)} \\ \left[1 - 0.22 \cdot \left(\frac{M_{crd}(L)}{M_y}\right)^{0.5}\right]\left(\frac{M_{crd}(L)}{M_y}\right)^{0.5} \cdot M_y \quad \text{if } \lambda_d(L) > 0.673 & \text{(Eq. 1.2.2-9)} \end{cases}$$

Predicted flexural strength per DSM 1.3

$$M_{n2}(L) := \min\left(\left(M_{ne2}(L) \quad M_{nl2}(L) \quad M_{nd2}(L)\right)\right)$$

Developed beam chart compared with modified section

DSM beam chart for the 9CS2.5x059 and the modified section



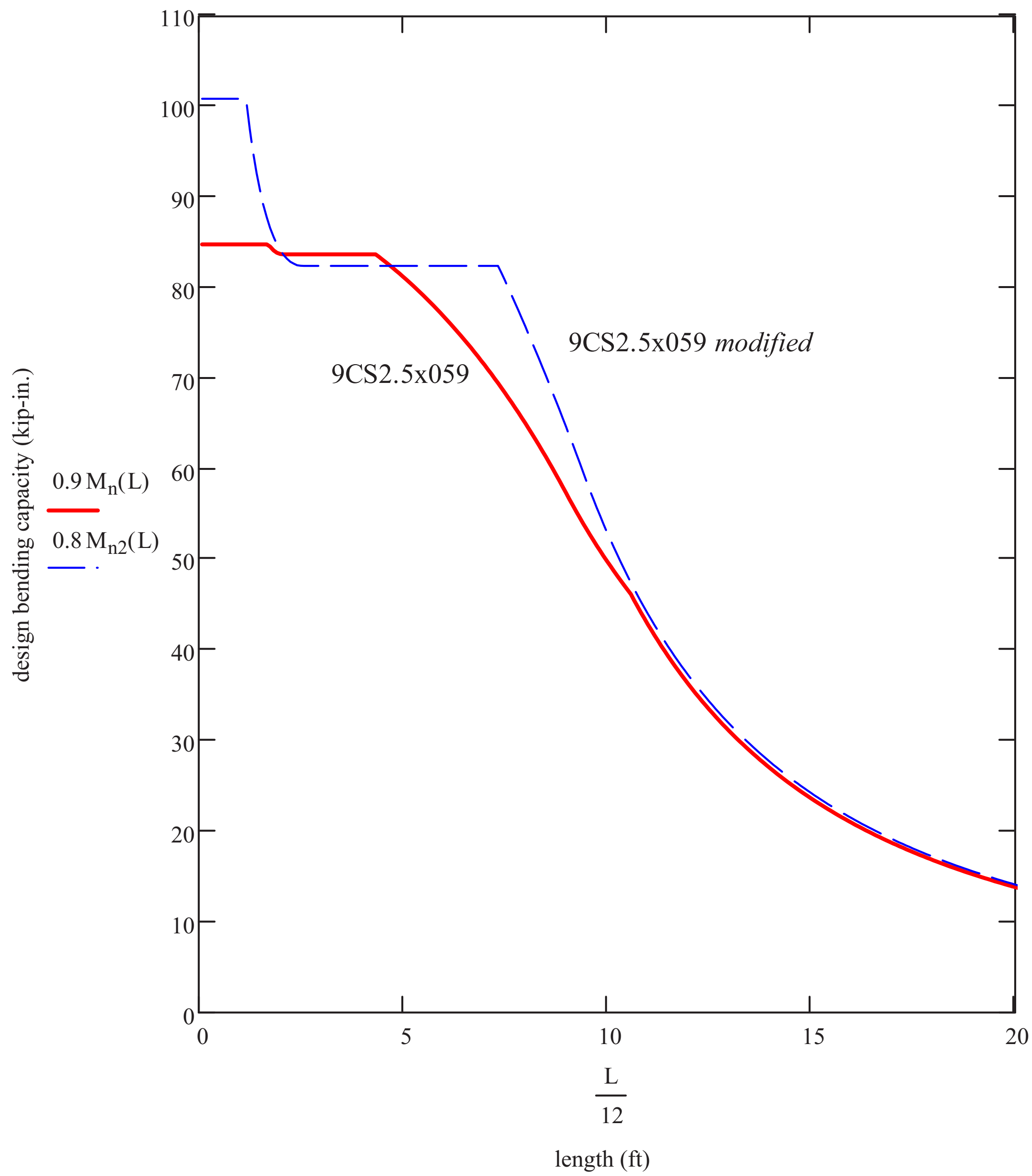
Notes on the beam chart:

(1) The modified section has higher capacity than the original section for all lengths

(2) Although local buckling is increased greatly for the modified section (note strength at L=0) distortional buckling controls over a much larger range for the modified section, e.g., from approximately 2 to 9 ft.

Developed beam chart compared with modified section (with resistance factors)

DSM beam chart for the 9CS2.5x059 and the modified section



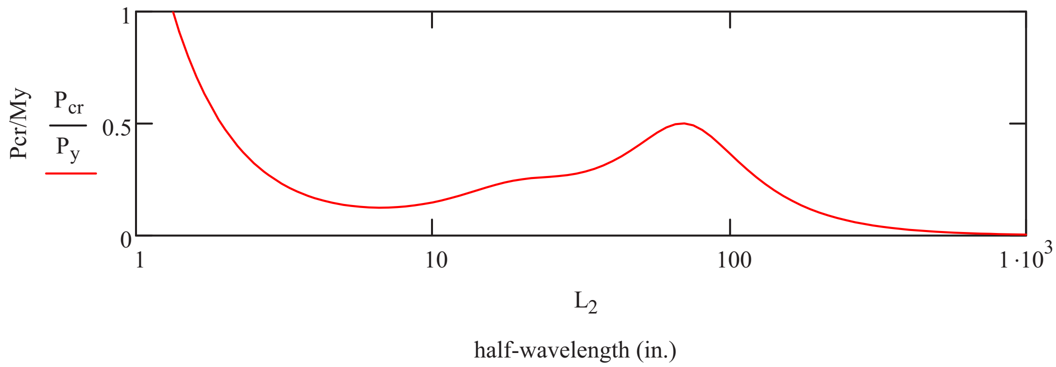
Notes on the beam chart:

(1) The lower resistance factor for the modified cross- section reduces the advantage of the modification, and over a short range of lengths actually provides less design strength than the original cross-section.

(2) In the modified cross-section distortional buckling controls over a much larger length than in the original cross-section.

8.14 Development of a column chart for C-section with lips (Example 3.2.1)

Consider again the C-section (9CS2.5x059) with the now familiar elastic bucking analysis curve



The finite strip analysis enforces a single half sine wave for the deformation along the length to develop the above figure. To develop a column chart one must determine how each of the buckling modes will behave without this restriction. That is to say, find $P_{cr\ell}$, P_{crd}, P_{cre} as a function of length.

Local buckling ($P_{cr\ell}$) as a function of length

For lengths longer than 7 in. (the local buckling minimum in the above figure) local buckling will simply repeat itself undulating up and down. Length shorter than 7 in. is not of interest, so it is assumed that the local buckling value is constant with length.

$P_{crlp} := 0.12 \cdot P_y$ local buckling minimum from FSM

$P_{crl}(L) := P_{crlp}$ local buckling does not change (even at short lengths)

Distortional buckling (P_{crd}) as a function of length

For lengths longer than 28 in. (the distortional buckling minimum in the above figure) distortional buckling will simply repeat with multiple half-waves along the length. For short columns the increase in distortional buckling as it is restricted may be worthy of investigation. The closed-form solution of Section 2.6 could be used, but the simpler empirical expression given below has been found to be adequate.

$P_{crdp} := 0.27 \cdot P_y$ $L_{crd} := 28.5$ distortional buckling minimum from FSM

$$P_{crd}(L) := \begin{cases} P_{crdp} & \text{if } L \ge L_{crd} \\ P_{crdp} \cdot \left(\frac{L}{L_{crd}}\right)^{\ln\left(\frac{L}{L_{crd}}\right)} & \text{if } L < L_{crd} \end{cases}$$

Global buckling (P_{cre}) as a function of length

Global buckling is strongly dependent on length. The closed-form solutions of the main *Specification* (as demonstrated in Chapter 9) can be used to provide P_{cre} as a function of length - but here an alternate approach that only employs the FSM analysis is used. This is convenient when quickly exploring the strength of different members and avoids recourse to the longer *Specification* equations and the need for dealing with section properties.

For this case the form of $P_{cre}{}^2 \sim= \alpha(1/L)^2 + \beta(1/L)^4$ if $L=L_x=L_t$

Pick two points along the global buckling curve to fit to

$L_{cr1} := L2_{70}$ $\quad P_{cre1} := \left(P_{cr_{70}}\right)$ $\quad L_{cr2} := L2_{90}$ $\quad P_{cre2} := \left(P_{cr_{90}}\right)$

Note, here the 70th and 90th points are selected from the FSM analysis. Any two points which are clearly in the global (lateral-torsional) buckling regime will do.

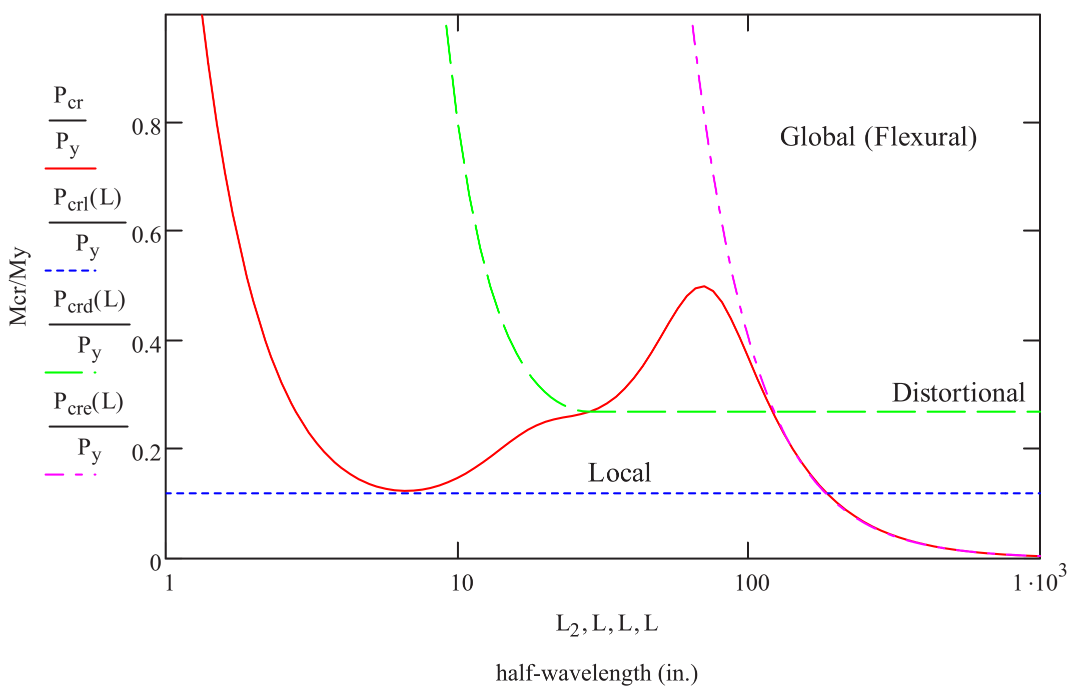
The constants α and β are found (two equations, two unknowns) via:

$$\alpha := \frac{P_{cre1}{}^2 \cdot L_{cr1}{}^4 - P_{cre2}{}^2 \cdot L_{cr2}{}^4}{L_{cr1}{}^2 - L_{cr2}{}^2} \qquad \beta := \frac{\left(P_{cre1}{}^2 \cdot L_{cr1}{}^2 - P_{cre2}{}^2 \cdot L_{cr2}{}^2\right) \cdot L_{cr1}{}^2 \cdot L_{cr2}{}^2}{L_{cr2}{}^2 - L_{cr1}{}^2}$$

$$P_{cre}(L) := \sqrt{\alpha \cdot \left(\frac{1}{L}\right)^2 + \beta \cdot \left(\frac{1}{L}\right)^4}$$

to identify the flexural mode as "flexural" (and not torsional-flexural) we must visually examine the mode shape.

Based on these assumptions the buckling loads as a function of length are:



Strength predictions, via DSM (Appendix 1 of AISI 2004)

Global buckling check per DSM 1.2.1.1

$$\lambda_c(L) := \sqrt{\frac{P_y}{P_{cre}(L)}}$$ note (L) denotes that the quantity is a function of length (Eq. 1.2.1-3)

$$P_{ne}(L) := \begin{cases} 0.658^{\lambda_c(L)^2} \cdot P_y & \text{if } \lambda_c(L) \le 1.5 \\ \dfrac{.877}{\lambda_c(L)^2} \cdot P_y & \text{if } \lambda_c(L) > 1.5 \end{cases}$$

(Eq. 1.2.1-1)
(Eq. 1.2.1-2)

Local buckling check per DSM 1.2.1.2

$$\lambda_l(L) := \sqrt{\frac{P_{ne}(L)}{P_{crl}(L)}}$$ (subscript "l" = "ℓ") (Eq. 1.2.1-7)

$$P_{nl}(L) := \begin{cases} P_{ne}(L) & \text{if } \lambda_l(L) \le 0.776 \\ \left[1 - 0.15 \cdot \left(\dfrac{P_{crl}(L)}{P_{ne}(L)}\right)^{0.4}\right]\left(\dfrac{P_{crl}(L)}{P_{ne}(L)}\right)^{0.4} \cdot P_{ne}(L) & \text{if } \lambda_l(L) > 0.776 \end{cases}$$

(Eq. 1.2.1-5)
(Eq. 1.2.1-6)

Distortional buckling check per DSM 1.2.1.3

$$\lambda_d(L) := \sqrt{\frac{P_y}{P_{crd}(L)}}$$ (Eq. 1.2.1-10)

$$P_{nd}(L) := \begin{cases} P_y & \text{if } \lambda_d(L) \le 0.561 \\ \left[1 - 0.25 \cdot \left(\dfrac{P_{crd}(L)}{P_y}\right)^{0.4}\right]\left(\dfrac{P_{crd}(L)}{P_y}\right)^{0.4} \cdot P_y & \text{if } \lambda_d(L) > 0.561 \end{cases}$$

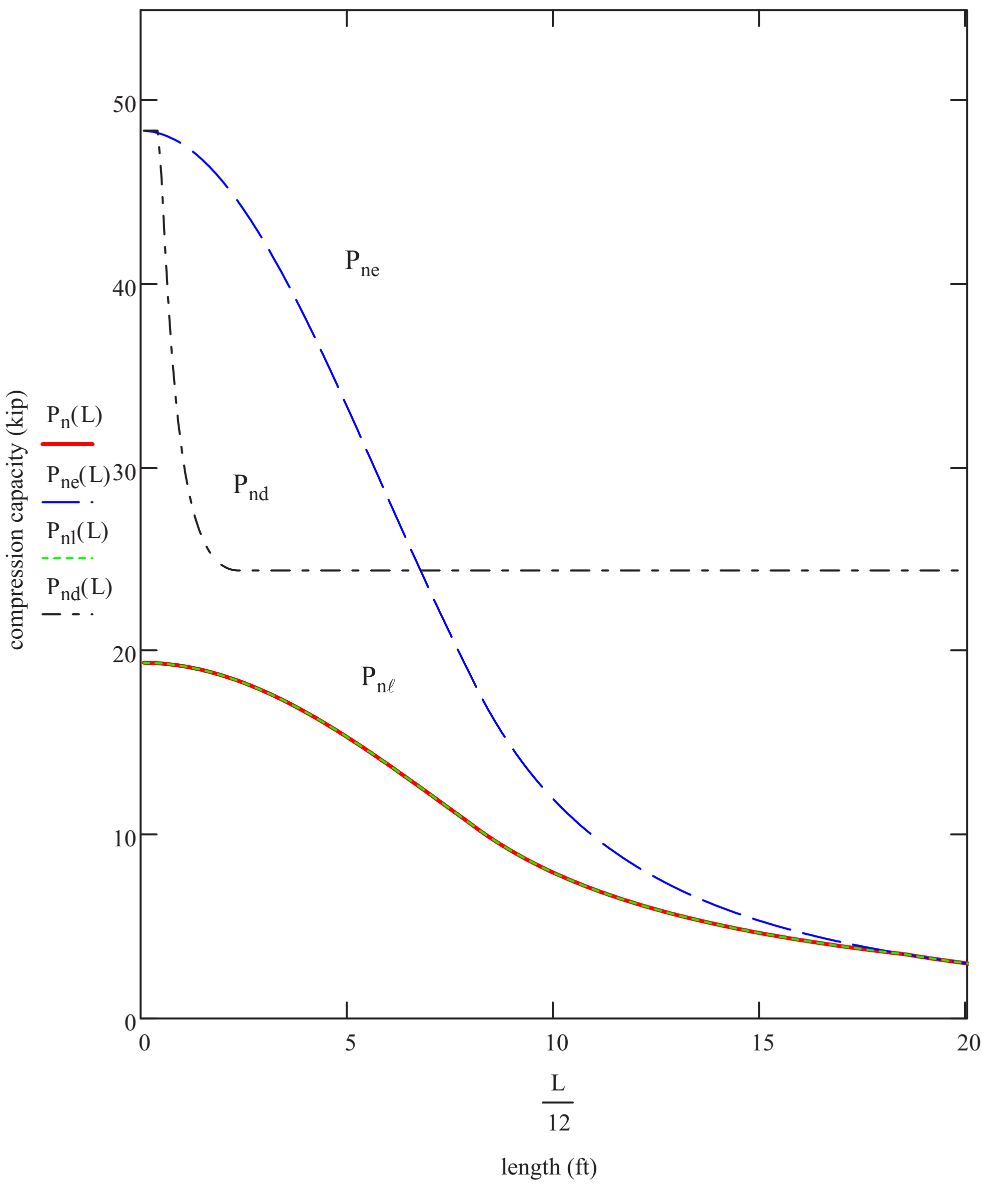
(Eq. 1.2.1-8)
(Eq. 1.2.1-9)

Predicted compressive strength per DSM 1.2

$$P_n(L) := \min\left(\left(P_{ne}(L) \quad P_{nl}(L) \quad P_{nd}(L)\right)\right)$$

Developed column chart

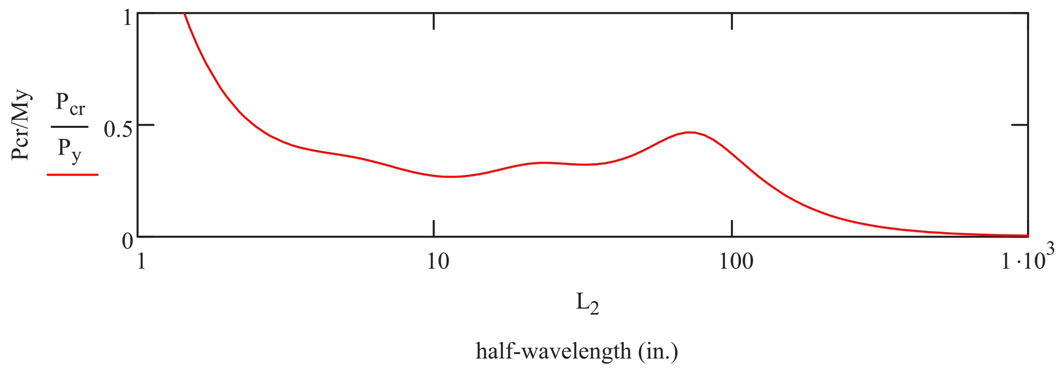
DSM column chart for the 9CS2.5x059



Notes on the column chart:
(1) Local buckling dominates the actual column strength
(2) The reduction due to local buckling is large even for short columns
(3) Distortional buckling never controls in this section

Column chart for C- section with lips *modified* (Section 3.2.2)

Consider again the *modified* C-section (9CS2.5x059) with the now familiar elastic bucking curve



The finite strip analysis enforces a single half sine wave for the deformation along the length. To develop a Column Chart one needs to determine how each of the buckling modes will behave without this restriction. That is to say, find $P_{cr\ell}$, P_{crd}, P_{cre} as a function of length.

Local buckling ($P_{cr\ell}$) as a function of length

For lengths longer than 11.5 in. (the local buckling minimum in the above figure) local buckling will simply repeat itself undulating up and down. Length shorter than 11.5 in. is not of interest, so it is assumed that the local buckling value is constant with length.

$P_{crlp} := 0.27 \cdot P_y$ local buckling minimum from FSM

$P_{crl}(L) := P_{crlp}$ local buckling is assumed invariant (even at short lengths)

Distortional buckling (P_{crd}) as a function of length

For lengths longer than 33 in. (the distortional buckling minimum in the above figure) distortional buckling will simply repeat with multiple half-waves along the length. For short columns the increase in distortional buckling as it is restricted may be worthy of investigation. The closed-form solution of section 2.5 could be used, but the simpler empirical expression given below has been found to be adequate.

$P_{crdp} := 0.32 \cdot P_y$ $L_{crd} := 32.7$ distortional buckling minimum from FSM

$$P_{crd}(L) := \begin{cases} P_{crdp} & \text{if } L \geq L_{crd} \\ P_{crdp} \cdot \left(\frac{L}{L_{crd}}\right)^{\ln\left(\frac{L}{L_{crd}}\right)} & \text{if } L < L_{crd} \end{cases}$$

Global buckling (P_{cre}) as a function of length

Global buckling is strongly dependent on length. The closed-form solutions of the main *Specification* (as illustrated in section 2.5) can be used to provide P_{cre} as a function of length - but here an alternate approach that only employs the FSM analysis is used. This is convenient when quickly exploring the strength of different members and avoids recourse to the longer Specification equations and the need for dealing with section properties.

For our case the form of $P_{cre}^2 \sim= \alpha(1/L)^2 + \beta(1/L)^4$ if $L=L_y=L_t$

Pick two points along the global buckling curve to fit to

$L_{cr1} := L2_{70}$ $P_{cre1} := \left(P_{cr_{70}}\right)$ $L_{cr2} := L2_{90}$ $P_{cre2} := \left(P_{cr_{90}}\right)$

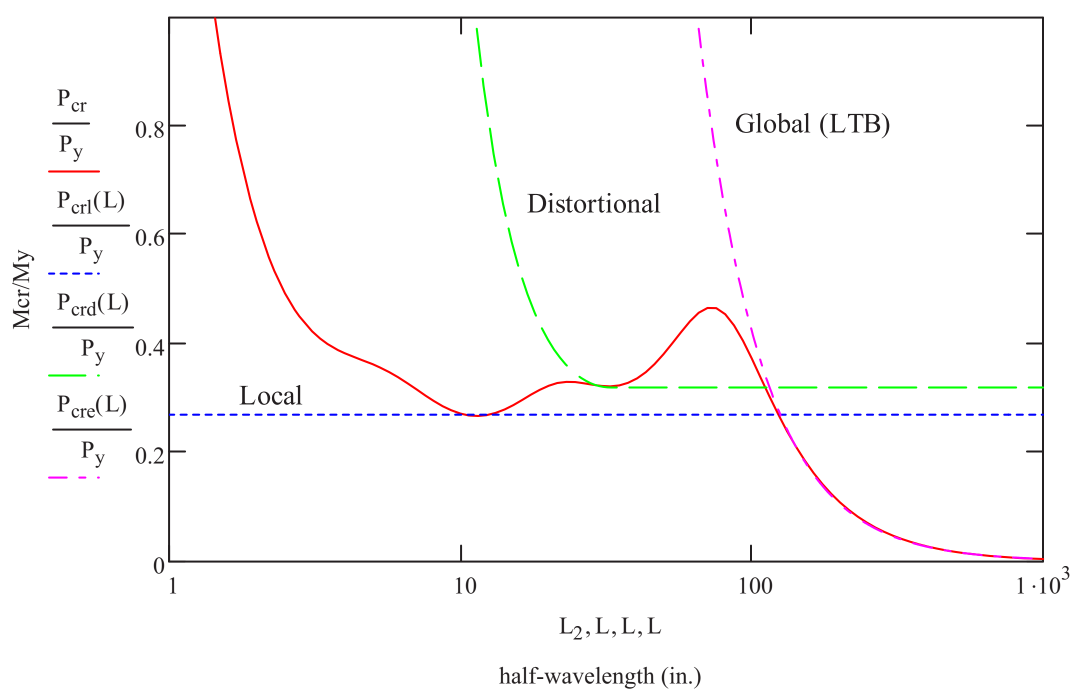
Note, here the 70th and 90th points are selected from the FSM analysis. Any two points which are clearly in the global (lateral-torsional) buckling regime will do.

The constants α and β are found (two equations, two unknowns) via:

$$\alpha := \frac{P_{cre1}^{\ 2} \cdot L_{cr1}^{\ 4} - P_{cre2}^{\ 2} \cdot L_{cr2}^{\ 4}}{L_{cr1}^{\ 2} - L_{cr2}^{\ 2}} \qquad \beta := \frac{\left(P_{cre1}^{\ 2} \cdot L_{cr1}^{\ 2} - P_{cre2}^{\ 2} \cdot L_{cr2}^{\ 2}\right) \cdot L_{cr1}^{\ 2} \cdot L_{cr2}^{\ 2}}{L_{cr2}^{\ 2} - L_{cr1}^{\ 2}}$$

$$P_{cre}(L) := \sqrt{\alpha \cdot \left(\frac{1}{L}\right)^2 + \beta \cdot \left(\frac{1}{L}\right)^4}$$

Based on these assumptions the buckling moments as a function of length:



Strength predictions, via DSM (Appendix 1 AISI 2004)

Global buckling check per DSM 1.2.1.1

$$\lambda_c(L) := \sqrt{\frac{P_y}{P_{cre}(L)}} \qquad \text{(Eq. 1.2.1-3)}$$

$$P_{ne2}(L) := \begin{cases} 0.658^{\lambda_c(L)^2} \cdot P_y & \text{if } \lambda_c(L) \le 1.5 \qquad \text{(Eq. 1.2.1-1)} \\ \dfrac{.877}{\lambda_c(L)^2} \cdot P_y & \text{if } \lambda_c(L) > 1.5 \qquad \text{(Eq. 1.2.1-2)} \end{cases}$$

Local buckling check per DSM 1.2.1.2

$$\lambda_l(L) := \sqrt{\frac{P_{ne2}(L)}{P_{crl}(L)}} \qquad \text{(subscript "l" = "}\ell\text{")} \qquad \text{(Eq. 1.2.1-7)}$$

$$P_{nl2}(L) := \begin{cases} P_{ne2}(L) & \text{if } \lambda_l(L) \le 0.776 \qquad \text{(Eq. 1.2.1-5)} \\ \left[1 - 0.15 \cdot \left(\dfrac{P_{crl}(L)}{P_{ne2}(L)}\right)^{0.4}\right]\left(\dfrac{P_{crl}(L)}{P_{ne2}(L)}\right)^{0.4} \cdot P_{ne2}(L) & \text{if } \lambda_l(L) > 0.776 \qquad \text{(Eq. 1.2.1-6)} \end{cases}$$

Distortional buckling check per DSM 1.2.1.3

$$\lambda_d(L) := \sqrt{\frac{P_y}{P_{crd}(L)}} \qquad \text{(Eq. 1.2.1-10)}$$

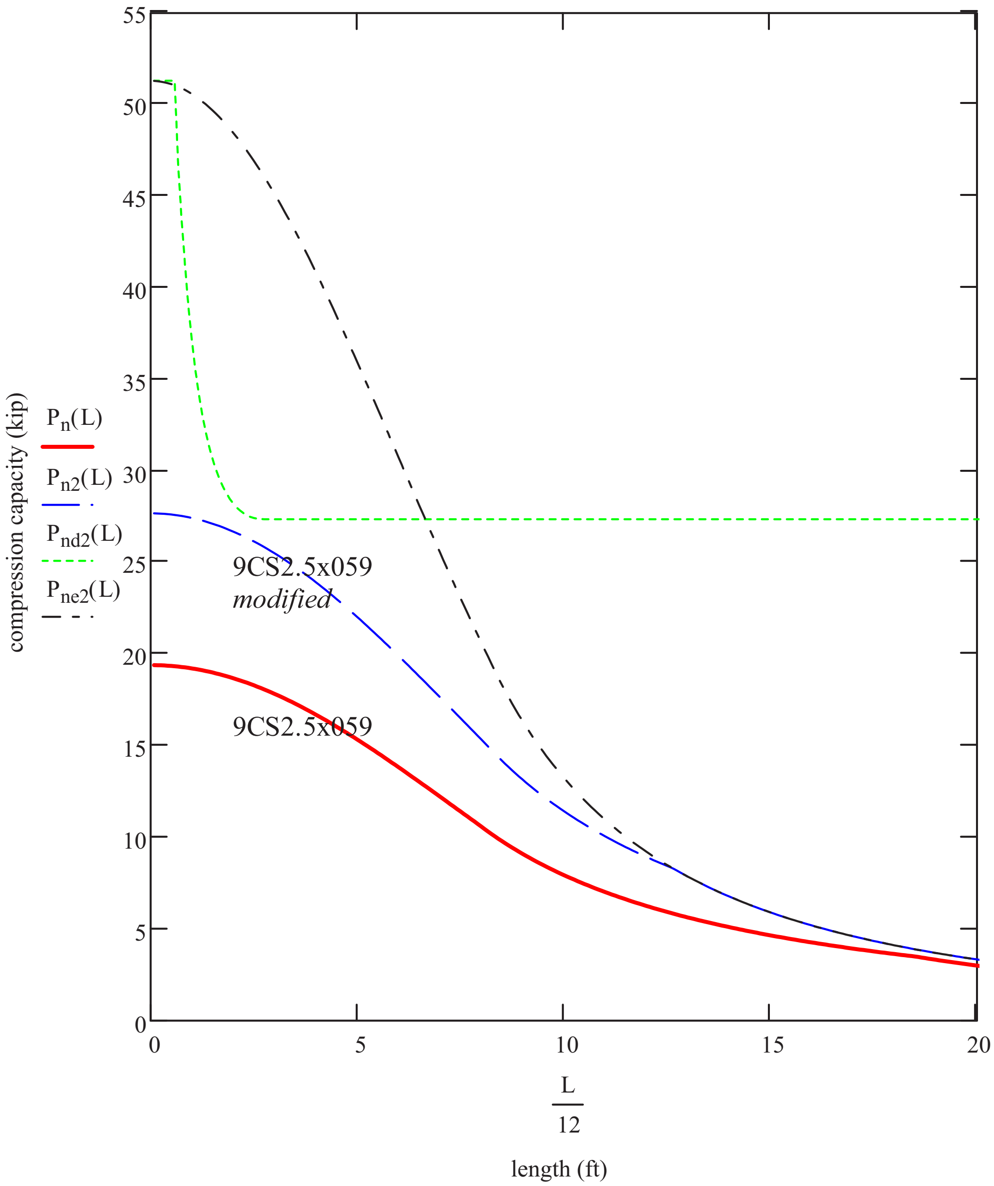
$$P_{nd2}(L) := \begin{cases} P_y & \text{if } \lambda_d(L) \le 0.561 \qquad \text{(Eq. 1.2.1-8)} \\ \left[1 - 0.25 \cdot \left(\dfrac{P_{crd}(L)}{P_y}\right)^{0.4}\right]\left(\dfrac{P_{crd}(L)}{P_y}\right)^{0.4} \cdot P_y & \text{if } \lambda_d(L) > 0.561 \qquad \text{(Eq. 1.2.1-9)} \end{cases}$$

Predicted compressive strength per DSM 1.2

$$P_{n2}(L) := \min\left(\left(P_{ne2}(L) \quad P_{nl2}(L) \quad P_{nd2}(L)\right)\right)$$

Column chart for the C-section with lips modified

Following the same procedure as for the standard C-section a DSM Column Chart for the 9CS2.5x059 *modified* is generated and compared to the original 9CS2.5x059 C-section below.



Notes on the column chart:

(1) Local buckling dominates the strength, but the modified section is much improved

(2) The modified section provides greater capacity at all lengths

(3) Distortional buckling never controls in either section, but further improvement to local buckling without improvement to distortional buckling will not benefit this section much, because distortional buckling will begin to control.

8.15 Comparison of DSM with the main *Specification*

A comparison of the main *Specification* strength prediction and those of DSM Appendix 1 is provided in Table 6 for the example problems of this Chapter. In some cases the main *Specification* predictions are not applicable (designated N/A in the table) due to the unique geometry of the modified cross-sections, in other cases the AISI (2002) *Design Manual* does not provide a specific calculation and thus, none is given here (designated "-" in the table).

Ease of calculation/modification: the design examples are intended to illustrate that when a numerical elastic buckling analysis tool such as CUFSM is available the Direct Strength Method requires less calculation and complexity than the main *Specification*. For example in the AISI (2002) *Design Manual*, presentation of the bending strength calculation for the C-section with lips takes $4^1/_2$ pages and intermediate iterations are still left out; calculation for the bending strength of the same C-section is presented in under 2 pages in Section 8.1 of this Guide. More importantly a significant modification, including the addition of web stiffeners, still takes less than 2 pages to present the bending strength prediction as shown in Section 8.2 of this Guide.

Slender elements: DSM discourages the use of very slender individual elements, as this drives the entire cross-section capacity down. For example, consider the compression capacity of the C-section with lips at F_n=37.25 ksi vs. the modified cross-section with web stiffeners added, as shown in Table 6: P_n=15.2 kips unmodified and 21.6 kips modified. This 42% improvement in strength comes solely from the addition of two small ¼ in. web stiffeners in the otherwise very slender web. When local buckling controls the strength small changes in the geometry are rewarded generously by DSM, less so by the main *Specification*.

Distortional buckling: the last column of Table 6 indicates which mode controlled the strength prediction in the DSM method, in a number of cases distortional buckling controls. The main *Specification* does not include an explicit check for distortional buckling, therefore it is anticipated that DSM may provide lower strength predictions in these cases. Indeed, where direct comparisons are available, this is the case: DSM predictions where distortional buckling controls are lower than those of the main *Specification*. If bracing partially restricts distortional buckling, which is sometimes the case in practice, this may be included in the elastic buckling analysis (see Section 3.3.7 of this Guide). In this situation M_{crd} or P_{crd} may be elevated to the point at which it no longer controls. Though not currently included, the addition of a distortional buckling provision to the main *Specification* is a future possibility, and would eliminate this difference.

Local buckling: DSM predicts that local buckling is more likely to control the strength in compression than in bending. This is due to the use of relatively slender webs in standard cross-sections. For cross-sections which are modified, or optimized, local buckling is generally less likely to control the fully braced strength in compression or bending, and instead distortional buckling controls. Modifications to increase local buckling are usually relatively simple, a corrugation or stiffener will do, while modifications for distortional buckling generally are more involved. For cross-sections where local-global interaction is considered through the use of applied stresses F_n less than F_y, local buckling often controls the strength. Even though DSM includes an explicit check on distortional buckling, and the main *Specification* does not. For many practical unbraced lengths local-global interaction governs the capacity and the main *Specification's* lack of a distortional buckling check is irrelevant.

Table 6 Comparison of the main *Specification* and DSM Appendix 1

	main *Specification*				DSM Appendix 1				mode
	M_n (kip-in.)	ϕM_n (kip-in.)	P_n (kip)	ϕP_n (kip)	M_n (kip-in.)	ϕM_n (kip-in.)	P_n (kip)	ϕP_n (kip)	
C-section with lips (9CS2.5x059)									
Major-axis bending capacity (fully braced)	104	94			93	84			D
Compression capacity (fully braced)			24.3	20.7			19.4	16.5	L
Compression capacity at F_n=37.25ksi			19.2	16.3			15.2	12.9	L
C-section with lips *modified*									
Major-axis bending capacity (fully braced)	N/A	N/A			103	82			D
Compression capacity (fully braced)			-	-			22.6	18.1	D
Compression capacity at 37.25ksi			-	-			21.6	17.2	L
C-section without lips (550T125-54)									
Major-axis bending capacity (fully braced)	-	-			19	15			D*
Major-axis bending capacity at F_n=30.93	20	18			19	15			D*
Compression capacity (fully braced)			9	7.65			7.1	5.7	D*
Minor-axis bending capacity (flange in comp.)	-	-			1.69	1.35			Y
Minor-axis bending capacity (flange in tens.)	-	-			1.69	1.35			Y
C-section without lips *modified*									
Major-axis bending capacity (fully braced)	N/A	N/A			22	18			D*
Compression capacity (fully braced)			N/A	N/A			8.0	6.4	D*
Z-section with lips (8ZS2.25x059)									
Restrained x-axis bending (fully braced)	98	88			76	69			D
Compression capacity (fully braced)			-	-			19	16.1	D
Compression capacity at F_n=25.9ksi			15.0	12.7			12.5	10.6	L
Z-section with lips *modified*									
Restrained x-axis bending (fully braced)	N/A	N/A			83	66			D
Compression capacity (fully braced)			-	-			20.5	16.4	D
Compression capacity at F_n=25.9ksi			-	-			15.1	12.1	L
Equal leg angle with lips (4LS4x060)									
Restrained x-axis bending (fully braced)	-	-			17	13			Y
Compression capacity (fully braced)			-	-			17.6	14	L
Compression capacity at F_n=14.7ksi			5.6	4.8			7.5	6	L
Equal leg angle (2LU2x060)									
Restrained x-axis bending (fully braced)	-	-			1.7	1.3			D*
Compression capacity (fully braced)			-	-			4.3	3.4	L
Compression capacity at F_n=12.0ksi			2.3	1.9			2.2	1.8	L
Hat section (3HU4.5x135)									
Minor axis bending capcity (web in comp)	76	68			79	71			Y
Compression capacity (fully braced)			87.0	74.0			86.8	69.5	Y
Wall panel									
bending (top flange in compression)	12.8	11.5			11.4	9.1			L
bending (bottom flange in compression)	12.4	11.2			10.7	8.5			D
Rack post section									
x-axis bending (fully braced)	-	-			19	15			D
z-axis bending (fully braced)			-	-	11	9			D
Compression capacity (fully braced)			-	-			15.7	12.5	D
Sigma section									
Major-axis bending capacity (fully braced)	N/A	N/A			71	57			D
Compression capacity (fully braced)			-	-			30.9	24.7	L
Compression capacity (braced at 66in.)			-	-			21	16.8	D

N/A = not applicable, calculation cannot be made via the rules of the main Specification without rational analysis extension
- = not provided as a calculation in the AISI (2002) Manual, though possible by main Specification
Y = section reaches yield capacity
L = local buckling strength equations control
D = distortional buckling strength controls
D* = distortional buckling strength controls, but equations have been conservatively extended to cover this section

Lip stiffeners and Specification versus DSM: the main *Specification* and DSM Appendix 1 do not give the same optimum cross-sections. For example, with regard to lip stiffener length, the main *Specification* generally discourages long lip stiffeners, and thus those used in practice are relatively short. However, DSM (supported by testing, e.g., see Schafer 2002) encourages significantly longer lip stiffeners and rewards these cross-sections with higher capacities, particularly with regard to distortional buckling.

Equal leg angle with lips: the DSM and main *Specification* predictions for the equal leg angle with lips, Section 3.2.7, provide fundamentally different results: P_n = 5.6 kips vs. 7.5 kips. This is due to the different ways that distortional buckling is handled for this cross-section. In the main *Specification* the edge stiffener provisions are invoked (a partial accounting for distortional buckling) and the effective width of the angle legs is reduced. In the DSM example problem, based on the finite strip analysis, it is recognized that distortional buckling of this cross-section is not distinct from torsional buckling, for a fully braced column the only reduction occurs from local buckling, not distortional. For a given unbraced length, the DSM method would include local-global interaction that would further reduce the capacity, but for a fully braced column, as computed in the example, no such reduction is relevant.

Long beams and columns: global buckling is not investigated in detail in this comparison since DSM and the main *Specification* use identical strength expressions for determining the global beam or column strength. For longer unbraced lengths this dominates the results and the two methods yield the same strength prediction. However, for intermediate unbraced lengths, DSM includes an explicit distortional buckling check, and handles local-global interaction in a different manner; as such, the predictions by the two methods can diverge in this range.

Rational analysis vs. pre-qualified: Many of the cross-sections analyzed with the Direct Strength Method do not meet the pre-qualified geometry, and as a result must use lower ϕ and higher Ω factors. This situation may tend to distort the optimal design, in the sense that the nominal strength may indicate that the addition of a web stiffener is highly beneficial, but the design strength does not – since the cross-section is no longer pre-qualified. Additional discussion on pre-qualified members can be found in Sections 1.3.2 and 7.4 of this Guide.

Z-section and Z-section modified: The main *Specification* provides strength predictions for Z-sections that are higher than those of the Direct Strength Method for a fully braced cross-section. This is primarily due to the inclusion of un-restrained distortional buckling in the DSM calculation. If distortional buckling is restrained the prediction by the two methods is similar.

As Table 6 shows, the main *Specification* and DSM (Appendix 1) provide similar, but different predictions for the strength. Since the methodologies are different, the optimal design (highest strength) that the two methods implicitly encourage is also different. DSM's advantages lie in the ease of calculation and the ability to confidently examine variations in cross-section, while the main *Specification* provides a more prescriptive design methodology.

9 Manual elastic buckling solutions

Instead of using FSM or other numerical methods to determine the elastic buckling loads (or moments), for C and Z or other common sections, formulas are referenced in the Commentary to Appendix 1 for manual elastic buckling calculation. An example for manual elastic buckling of a C section column and beam is provided in this Chapter.

Column solution ($P_{cr\ell}$, P_{crd}, P_{cre}) C-section with lips

Geometry:

Many of the developed hand solutions do not specifically include corner radii in the solution, instead centerline dimensions are employed. For this reason the examples given here for local and distortional buckling will employ centerline dimensions. The selected C-section with lips is a 9CS2.5x059 from the AISI *Cold-Formed Steel Design Manual* (2002).

Centerline Dimensions:

$t := 0.059 \cdot in$

$h := 9 \cdot in - t$ $\qquad h = 8.941\ in$

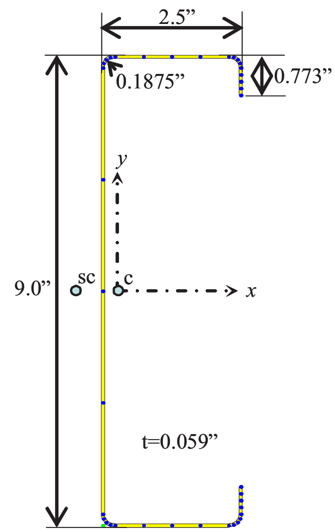
$b := 2.5 \cdot in - t$ $\qquad b = 2.441\ in$

$d := 0.773 \cdot in - \frac{t}{2}$ $\qquad d = 0.744\ in$

$\theta := 90 \cdot \frac{\pi}{180}$

$E := 29500 \cdot ksi$

$\nu := 0.3$



out-to-out dimensions shown in figure

Local Buckling ($P_{cr\ell}$)

The basic methodology for local buckling determinations is discussed in the Appendix 1 (AISI 2004) Commentary in Section 1.1.2.2 Elastic Buckling - Manual Solutions.

$P_{crl} := A_g \cdot f_{crl}$ $\qquad$ per C-1.1.2-1

$$f_{crl} := k \cdot \frac{\pi^2 \cdot E}{12 \cdot \left(1 - \nu^2\right)} \cdot \left(\frac{t}{w}\right)^2$$ per C-1.1.2-3

The discussion for determining k in the Appendix 1 Commentary suggests two methods that may be used (1) determine the buckling stress of the individual elements or (2) use semi-empirical equations that account for the interaction of any two elements. Method (1) is known as the *element method* and (2) as the *interaction method*. Both methods are illustrated here.

(continued) Local Buckling ($P_{cr\ell}$)

Element Method:

Flange Local Buckling:

Classical solution for a simply-supported plate in pure compression is employed.

$k_{flange} := 4$ (per Table C-B2-1)

$$f_{cr\_flange} := k_{flange} \cdot \frac{\pi^2 \cdot E}{12 \cdot \left(1 - \nu^2\right)} \cdot \left(\frac{t}{b}\right)^2 \qquad f_{cr\_flange} = 62.306\,\text{ksi}$$

Web Local Buckling:

Classical solution for a simply-supported plate in pure compression is employed.

$k_{web} := 4$ (per Table C-B2-1)

$$f_{cr\_web} := k_{web} \cdot \frac{\pi^2 \cdot E}{12 \cdot \left(1 - \nu^2\right)} \cdot \left(\frac{t}{h}\right)^2 \qquad f_{cr\_web} = 4.644\,\text{ksi}$$

Lip Local Buckling:

Classical solution for a plate simply-supported on three sides and free along one edge is employed.

$k_{lip} := 0.425$ (per Table C-B2-1)

$$f_{cr\_lip} := k_{lip} \cdot \frac{\pi^2 \cdot E}{12 \cdot \left(1 - \nu^2\right)} \cdot \left(\frac{t}{d}\right)^2 \qquad f_{cr\_lip} = 71.356\,\text{ksi}$$

$$f_{crl} := \min\left(\left(f_{cr\_flange} \;\; f_{cr\_web} \;\; f_{cr\_lip}\right)\right) \qquad f_{crl} = 4.644\,\text{ksi}$$

$$A_g := (h + 2 \cdot b + 2 \cdot d) \cdot t \qquad A_g = 0.903\,\text{in}^2$$

(using centerline dimensions)

$$P_{crl} := A_g \cdot f_{crl} \qquad P_{crl} = 4.195\,\text{kip}$$

(continued) Local Buckling ($P_{cr\ell}$)

Interaction Method:

Flange / Lip Local Buckling

This expression for k, is given in Schafer (2002). The expression is based on an empirical curve fit to finite strip analysis of an isolated flange and lip. The expression accounts for the beneficial affect of the lip on the flange at intermediate lip lengths and also accounts for the detrimental affect of the lip on the flange at long lip lengths.

$$k_{flange\_lip} := -11.07 \cdot \left(\frac{d}{b}\right)^2 + 3.95 \cdot \left(\frac{d}{b}\right) + 4 \qquad k_{flange\_lip} = 4.176$$

Note, d/b should be less than 0.6 for this empirical expression to be applicable

$\frac{d}{b} = 0.305$ $\quad < 0.6$, therefore OK

$$f_{cr\_flange\_lip} := k_{flange\_lip} \cdot \frac{\pi^2 \cdot E}{12 \cdot \left(1 - \nu^2\right)} \cdot \left(\frac{t}{b}\right)^2 \qquad f_{cr\_flange\_lip} = 65.049\ \text{ksi}$$

Flange / Web Local Buckling

This expression for k is given in Schafer (2002). The expression is based on an empirical curve fit to finite strip analysis of an isolated flange and web. If h/b = 1 The k value is 4. If h/b > 1 the k value is reduced from 4 due to the buckling of the web. If h/b < 1 the k value is increased from 4 due to the restraint provided by the web to the flange.

$$k_{flange\_web} := \begin{cases} \left[2 - \left(\frac{b}{h}\right)^{0.4}\right] \cdot 4 \cdot \left(\frac{b}{h}\right)^2 & \text{if } \frac{h}{b} \ge 1 \\ \left[2 - \left(\frac{h}{b}\right)^{0.2}\right] \cdot 4 & \text{if } \frac{h}{b} < 1 \end{cases}$$

$\frac{h}{b} = 3.663$

$k_{flange\_web} = 0.419$

$$f_{cr\_flange\_web} := k_{flange\_web} \cdot \frac{\pi^2 \cdot E}{12 \cdot \left(1 - \nu^2\right)} \cdot \left(\frac{t}{b}\right)^2 \qquad f_{cr\_flange\_web} = 6.525\ \text{ksi}$$

$$f_{crl2} := \min\left(\left(f_{cr\_flange\_lip} \quad f_{cr\_flange\_web}\right)\right) \qquad f_{crl2} = 6.525\ \text{ksi}$$

$$P_{crl2} := A_g \cdot f_{crl2} \qquad P_{crl2} = 5.894\ \text{kip}$$

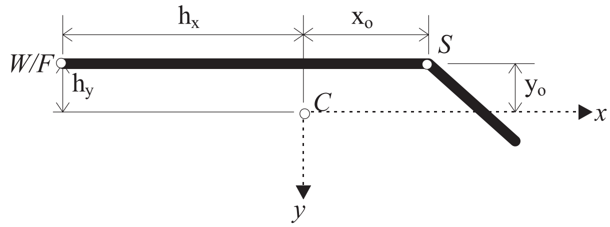
In this example local buckling is dominated by the slender web, but including the web/flange interaction is important as it increases the predicted buckling stress by 40%.

Distortional Buckling (P_{crd})

The basic methodology for distortional buckling determination is discussed in the Appendix 1 (AISI 2004) Commentary in Section 1.1.2.2 Elastic Buckling - Manual Solutions.

$$P_{crd} := A_g \cdot f_{crd} \qquad \text{per C-1.1.2-3}$$

The method illustrated here is based on Schafer (2002). Section properties of the isolated flange must be calculated. The expressions here are only applicable for simple lips. More complicated flanges would follow the same procedure, but new expressions would be required.



Material Properties:

$$G := \frac{E}{2 \cdot (1 + \nu)}$$

Properties of the Flange Only:

$$A_f := (b + d) \cdot t \qquad A_f = 0.188\ in^2$$

$$J_f := \frac{1}{3} \cdot b \cdot t^3 + \frac{1}{3} \cdot d \cdot t^3 \qquad J_f = 2.18 \times 10^{-4}\ in^4$$

$$I_{xf} := \frac{t \cdot \left(t^2 \cdot b^2 + 4 \cdot b \cdot d^3 - 4 \cdot b \cdot d^3 \cdot \cos(\theta)^2 + t^2 \cdot b \cdot d + d^4 - d^4 \cdot \cos(\theta)^2\right)}{12 \cdot (b + d)} \qquad I_{xf} = 6.709 \times 10^{-3}\ in^4$$

$$I_{yf} := \frac{t \cdot \left(b^4 + 4 \cdot d \cdot b^3 + 6 \cdot d^2 \cdot b^2 \cdot \cos(\theta) + 4 \cdot d^3 \cdot b \cdot \cos(\theta)^2 + d^4 \cdot \cos(\theta)^2\right)}{12 \cdot (b + d)} \qquad I_{yf} = 0.122\ in^4$$

$$I_{xyf} := \frac{t \cdot b \cdot d^2 \cdot \sin(\theta) \cdot (b + d \cdot \cos(\theta))}{4 \cdot (b + d)} \qquad I_{xyf} = 0.015\ in^4$$

$$I_{of} := \frac{t \cdot b^3}{3} + \frac{b \cdot t^3}{12} + \frac{t \cdot d^3}{3} \qquad I_{of} = 0.294\ in^4$$

$$x_{of} := \frac{b^2 - d^2 \cdot \cos(\theta)}{2 \cdot (b + d)}$$ x distance from the centroid to the shear center. $x_{of} = 0.936\ in$

$$y_{of} := \frac{-d^2 \cdot \sin(\theta)}{2 \cdot (b + d)}$$ y distance from the centroid to the shear center. $y_{of} = -0.087\ in$

$$h_{xf} := \frac{-\left(b^2 + 2 \cdot d \cdot b + d^2 \cdot \cos(\theta)\right)}{2(b + d)}$$ x distance from the centroid to the web/flange juncture. $h_{xf} = -1.505\ in$

$$h_{yf} := \frac{-d^2 \cdot \sin(\theta)}{2 \cdot (b + d)}$$ y distance from the centroid to the web/flange juncture. $h_{yf} = -0.087\ in$

$$C_{wf} := 0 \cdot in^6 \qquad C_{wf} = 0\ in^6$$

(continued) Distortional Buckling (P_{crd})

Determine the critical half-wavelength at which distortional buckling occurs:

$$L_{cr} := \left[\frac{6 \cdot \pi^4 \cdot h \cdot \left(1 - \nu^2\right)}{t^3} \cdot \left[I_{xf} \cdot \left(x_{of} - h_{xf}\right)^2 + C_{wf} - \frac{I_{xyf}^2}{I_{yf}} \cdot \left(x_{of} - h_{xf}\right)^2 \right] \right]^{\frac{1}{4}} \qquad L_{cr} = 28.52 \text{ in}$$

If bracing is provided that restricts the distortional mode at some length less than L_{cr}, then this length should be used in place of L_{cr}.

Determine the elastic and "geometric" rotational spring stiffness of the flange:

$$k_{\phi fe} := \left(\frac{\pi}{L_{cr}}\right)^4 \cdot \left[E \cdot I_{xf} \cdot \left(x_{of} - h_{xf}\right)^2 + E \cdot C_{wf} - E \cdot \frac{I_{xyf}^2}{I_{yf}} \cdot \left(x_{of} - h_{xf}\right)^2 \right] + \left(\frac{\pi}{L_{cr}}\right)^2 \cdot G \cdot J_f$$

$$k_{\phi fe} = 0.154 \text{ kip}$$

$$k_{\phi fg} := \left(\frac{\pi}{L_{cr}}\right)^2 \cdot \left[A_f \cdot \left[\left(x_{of} - h_{xf}\right)^2 \cdot \left(\frac{I_{xyf}}{I_{yf}}\right)^2 - 2 \cdot y_{of} \cdot \left(x_{of} - h_{xf}\right) \cdot \left(\frac{I_{xyf}}{I_{yf}}\right) + h_{xf}^2 + y_{of}^2 \right] + I_{xf} + I_{yf} \right]$$

$$k_{\phi fg} = 7.076 \times 10^{-3} \text{ in}^2$$

Determine the elastic and "geometric" rotational spring stiffness of the web:

$$k_{\phi we} := \frac{E \cdot t^3}{6 \cdot h \cdot \left(1 - \nu^2\right)} \qquad k_{\phi we} = 0.124 \text{ kip}$$

$$k_{\phi wg} := \left(\frac{\pi}{L_{cr}}\right)^2 \cdot \frac{t \cdot h^3}{60} \qquad k_{\phi wg} = 8.528 \times 10^{-3} \text{ in}^2$$

Determine the distortional buckling stress:

$$f_{crd} := \frac{k_{\phi fe} + k_{\phi we}}{k_{\phi fg} + k_{\phi wg}}$$

$$f_{crd} = 17.83 \text{ ksi}$$

$$P_{crd} := A_g \cdot f_{crd} \qquad P_{crd} = 16.106 \text{ kip}$$

Global Buckling (P_{cre})

Global cross-section properties are given for this section in AISI *Design Manual* (2002), Example I-1: C-Section With Lips - Gross Section Properties

$A := 0.881 \cdot in^2$ $\quad I_x := 10.3 \cdot in^4$ $\quad I_y := 0.698 \cdot in^4$

$J := 0.00102 \cdot in^4$ $\quad C_w := 11.9 \cdot in^6$ $\quad x_o := -1.66 \cdot in$

$r_x := \sqrt{\frac{I_x}{A}}$ $\quad r_x = 3.419\ in$ $\quad r_y := \sqrt{\frac{I_y}{A}}$ $\quad r_y = 0.89\ in$

$r_o := \sqrt{r_x^2 + r_y^2 + x_o^2}$ $\quad r_o = 3.904\ in$ (Eq. C3.1.2.1-12)

$\beta := 1 - \left(\frac{x_o}{r_o}\right)^2$ $\quad \beta = 0.819$ (Eq. C4.2-3)

Length and bracing conditions (for example)

$K_x := 1$ $\quad K_y := 1$ $\quad K_t := 1$

$L_x := 8 \cdot ft$ $\quad L_y := 8 \cdot ft$ $\quad L_t := 8 \cdot ft$

Individual buckling modes, per *Specification* C3.1.2

$\sigma_{ex} := \frac{\pi^2 \cdot E}{\left(\frac{K_x \cdot L_x}{r_x}\right)^2}$ (Eq. C3.1.2.1-7) $\quad \sigma_{ey} := \frac{\pi^2 \cdot E}{\left(\frac{K_y \cdot L_y}{r_y}\right)^2}$ (Eq. C3.1.2.1-8)

$\sigma_{ex} = 369.352\ ksi$ $\quad \sigma_{ey} = 25.03\ ksi$

$\sigma_t := \frac{1}{A \cdot r_o^2} \cdot \left[G \cdot J + \frac{\pi^2 \cdot E \cdot C_w}{(K_t \cdot L_t)^2}\right]$ $\quad \sigma_t = 28.864\ ksi$ (Eq. C3.1.2.1-9)

Torsional-flexural buckling per *Specification* C4.2

$F_e := \frac{1}{2 \cdot \beta} \cdot \left[(\sigma_{ex} + \sigma_t) - \sqrt{(\sigma_{ex} + \sigma_t)^2 - 4 \cdot \beta \cdot \sigma_{ex} \cdot \sigma_t}\right]$ $\quad F_e = 28.435\ ksi$ (Eq. C4.2-1)

Note, for singly symmetric sections the x-axis is assumed to be the axis of symmetry

$F_{e2} := \sigma_{ey}$ *Note*, F_{e2} is compared with F_e to determine the minimum buckling stress per the note in C4.2. If r_y is the least radius of gyration then F_{e2} is defined as given, otherwise F_{e2} should be calculated per the least radius of gyration, per C4.1.

$F_{cre} := \min((F_e\ \ F_{e2}))$ $\quad F_{cre} = 25.03\ ksi$

$P_{cre} := A \cdot F_{cre}$ $\quad P_{cre} = 22.051\ kip$

Beam solution ($M_{cr\ell}$, M_{crd}, M_{cre})

C-section with lips

Local Buckling (M_{crl})

The basic methodology for local buckling is discussed in the Appendix 1 Commentary in Section 1.1.2.2 Elastic Buckling - Manual Solutions.

$M_{crl} := S_g \cdot f_{crl}$ per C-1.1.2-2 where $S_g = I_x/c$ and $c := \frac{h}{2} + \frac{t}{2}$ $c = 4.5\,\text{in}$

$$f_{crl} := k \cdot \frac{\pi^2 \cdot E}{12 \cdot \left(1 - \nu^2\right)} \cdot \left(\frac{t}{w}\right)^2$$ per C-1.1.2-3

For local buckling k may be determined by the *element method* or the *interaction method*. Both methods are illustrated here.

Element Method:

Flange Local Buckling:

Classical solution for a simply-supported plate in pure compression is employed.

$k_{flange} := 4$ (per Table C-B2-1)

$$f_{cr\_flange} := k_{flange} \cdot \frac{\pi^2 \cdot E}{12 \cdot \left(1 - \nu^2\right)} \cdot \left(\frac{t}{b}\right)^2 \qquad f_{cr\_flange} = 62.306\,\text{ksi}$$

Web Local Buckling:

Classical solution for a simply-supported plate in pure bending is employed.

$k_{web} := 23.9$ (per Table C-B2-1)

$$f_{cr\_web} := k_{web} \cdot \frac{\pi^2 \cdot E}{12 \cdot \left(1 - \nu^2\right)} \cdot \left(\frac{t}{h}\right)^2 \qquad f_{cr\_web} = 27.748\,\text{ksi}$$

Lip Local Buckling:

Based on the work in Schafer and Pekoz (1999) the influence of the stress gradient on the lip can be accounted for (otherwise use k=0.425 and ignore the stress gradient on the lip).

$f_1 := 1$ stress at the extreme fiber $f_2 := \frac{c - d}{c}$ $f_2 = 0.835$ stress at end of lip

$\xi := \frac{f_1 - f_2}{f_1}$ $\xi = 0.165$ $k_{lip} := 1.4 \cdot \xi^2 - 0.25 \cdot \xi + 0.425$ $k_{lip} = 0.422$

clearly in this case the stress gradient on the lip is of little significance as k~0.425

$$f_{cr\_lip} := k_{lip} \cdot \frac{\pi^2 \cdot E}{12 \cdot \left(1 - \nu^2\right)} \cdot \left(\frac{t}{d}\right)^2 \qquad f_{cr\_lip} = 70.838\,\text{ksi}$$

(continued) Local Buckling ($M_{cr\ell}$)

(continued) Element method

$$f_{crl} := \min\left(\left(f_{cr\_flange} \quad f_{cr\_web} \quad f_{cr\_lip}\right)\right) \qquad f_{crl} = 27.748\,\text{ksi}$$

$$S_g := \frac{I_x}{c} \qquad S_g = 2.289\,\text{in}^3$$

$$M_{crl} := S_g \cdot f_{crl} \qquad M_{crl} = 63.512\,\text{kip} \cdot \text{in}$$

Interaction method

Flange / Lip Local Buckling

This expression for k, is given in Schafer and Pekoz (1999). The expression is based on an empirical curve fit to finite strip analysis of an isolated flange and lip. The expression accounts for the beneficial affect of the lip on the flange at intermediate lip lengths and also accounts for the detrimental affect of the lip on the flange at long lip lengths.

$f_1 := 1$ stress at the extreme fiber $\qquad f_2 := \frac{c - d}{c} \qquad f_2 = 0.835$ stress at end of lip

$$\xi := \frac{f_1 - f_2}{f_1} \qquad \xi = 0.165$$

$$k_{flange\_lip} := (8.55 \cdot \xi - 11.07) \cdot \left(\frac{d}{b}\right)^2 + (3.95 - 1.59 \cdot \xi) \cdot \left(\frac{d}{b}\right) + 4 \qquad k_{flange\_lip} = 4.227$$

Note, d/b should be less than 0.6 and $\xi<1$ for this empirical expression to be applicable

$\frac{d}{b} = 0.305$ $\quad < 0.6$, therefore OK $\qquad \xi = 0.165$ $\quad < 1.0$, therefore OK

$$f_{cr\_flange\_lip} := k_{flange\_lip} \cdot \frac{\pi^2 \cdot E}{12 \cdot \left(1 - \nu^2\right)} \cdot \left(\frac{t}{b}\right)^2 \qquad f_{cr\_flange\_lip} = 65.844\,\text{ksi}$$

Flange / Web Local Buckling

This expression for k is given in Schafer and Pekoz (1999). The expression is based on an empirical curve fit to finite strip analysis of an isolated flange and web.

stress gradient on the web $\qquad f_1 := 1 \qquad f_2 := -1 \qquad \xi := \frac{f_1 - f_2}{f_1} \qquad \xi = 2$

$$k_{flange\_web} := 1.125 \cdot \min\left[\left[4 \quad \left(0.5 \cdot \xi^3 + 4 \cdot \xi^2 + 4\right) \cdot \left(\frac{b}{h}\right)^2\right]\right] \qquad k_{flange\_web} = 2.012$$

$$f_{cr\_flange\_web} := k_{flange\_web} \cdot \frac{\pi^2 \cdot E}{12 \cdot \left(1 - \nu^2\right)} \cdot \left(\frac{t}{b}\right)^2 \qquad f_{cr\_flange\_web} = 31.347\,\text{ksi}$$

$$f_{crl2} := \min\left(\left(f_{cr\_flange\_lip} \quad f_{cr\_flange\_web}\right)\right) \qquad f_{crl2} = 31.347\,\text{ksi}$$

$$M_{crl2} := S_g \cdot f_{crl2} \qquad M_{crl2} = 71.75\,\text{kip} \cdot \text{in}$$

In this example local buckling is controlled by the slender web, but including the web/flange interaction is important as it increases the predicted buckling stress by 13%.

Distortional Buckling (M_{crd})

Determine the critical half-wavelength at which distortional buckling occurs:

$$L_{cr} := \left[\frac{4 \cdot \pi^4 \cdot h \cdot \left(1 - \nu^2\right)}{t^3} \cdot \left[I_{xf} \cdot \left(x_{of} - h_{xf}\right)^2 + C_{wf} - \frac{I_{xyf}^{\ 2}}{I_{yf}} \cdot \left(x_{of} - h_{xf}\right)^2\right] + \frac{\pi^4 \cdot h^4}{720}\right]^{\frac{1}{4}} \qquad L_{cr} = 25.783\ in$$

If bracing is provided that restricts the distortional mode at some length less than L_{cr}, then this length should be used in place of L_{cr}.

Determine the elastic and "geometric" rotational spring stiffness of the flange:

$$k_{\phi fe} := \left(\frac{\pi}{L_{cr}}\right)^4 \cdot \left[E \cdot I_{xf} \cdot \left(x_{of} - h_{xf}\right)^2 + E \cdot C_{wf} - E \cdot \frac{I_{xyf}^{\ 2}}{I_{yf}} \cdot \left(x_{of} - h_{xf}\right)^2\right] + \left(\frac{\pi}{L_{cr}}\right)^2 \cdot G \cdot J_f$$

$$k_{\phi fe} = 0.223\ kip$$

$$k_{\phi fg} := \left(\frac{\pi}{L_{cr}}\right)^2 \cdot \left[A_f \cdot \left[\left(x_{of} - h_{xf}\right)^2 \cdot \left(\frac{I_{xyf}}{I_{yf}}\right)^2 - 2 \cdot y_{of} \cdot \left(x_{of} - h_{xf}\right) \cdot \left(\frac{I_{xyf}}{I_{yf}}\right) + h_{xf}^{\ 2} + y_{of}^{\ 2}\right] + I_{xf} + I_{yf}\right]$$

$$k_{\phi fg} = 8.658 \times 10^{-3}\ in^2$$

Determine the elastic and "geometric" rotational spring stiffness of the web:

$$k_{\phi we} := \frac{E \cdot t^3}{12 \cdot \left(1 - \nu^2\right)} \cdot \left[\frac{3}{h} + \left(\frac{\pi}{L_{cr}}\right)^2 \cdot \frac{19 \cdot h}{60} + \left(\frac{\pi}{L_{cr}}\right)^4 \cdot \frac{h^3}{240}\right] \qquad k_{\phi we} = 0.21\ kip$$

stress gradient on the web $\quad f_1 := 1 \quad f_2 := -1 \quad \psi := \frac{f_1}{f_2} \quad \psi = -1$ (note ψ=1-ξ and ξ was used in the earlier stress gradient definitions)

$$k_{\phi wg} := \frac{h \cdot t \cdot \pi^2}{13440} \cdot \frac{\left(45360 \cdot \psi + 62160\right) \cdot \left(\frac{L_{cr}}{h}\right)^2 + 448 \cdot \pi^2 + \left(\frac{h}{L_{cr}}\right)^2 \cdot \left(53 + 3 \cdot \psi\right) \cdot \pi^4}{\pi^4 + 28 \cdot \pi^2 \cdot \left(\frac{L_{cr}}{h}\right)^2 + 420 \cdot \left(\frac{L_{cr}}{h}\right)^4}$$

$$k_{\phi wg} = 1.783 \times 10^{-3}\ in^2$$

Determine the distortional buckling stress:

$$f_{crd} := \frac{k_{\phi fe} + k_{\phi we}}{k_{\phi fg} + k_{\phi wg}}$$

$$f_{crd} = 41.41\ ksi$$

$$M_{crd} := S_g \cdot f_{crd} \qquad M_{crd} = 94.784\ kip \cdot in$$

Global buckling (M_{cre})

Global cross-section properties are given for this section in AISI *Design Manual* (2002), Example I-1: C-Section With Lips - Gross Section Properties and are listed above under column: global buckling.

Length and bracing conditions (for example)

$K_y := 1$ $\quad$ $K_t := 1$ $\quad$ $C_b := 1$ (simplification of C3.1.2.1-10)

$L_y := 10 \cdot ft$ $\quad$ $L_t := 10 \cdot ft$

Individual buckling modes, per C3.1.2.1(a)

$$\sigma_{ey} := \frac{\pi^2 \cdot E}{\left(\frac{K_y \cdot L_y}{r_y}\right)^2} \qquad \sigma_{ey} = 16.019\,ksi \qquad \text{(Eq. C3.1.2.1-8)}$$

$$\sigma_t := \frac{1}{A \cdot r_o^2} \cdot \left[G \cdot J + \frac{\pi^2 \cdot E \cdot C_w}{(K_t \cdot L_t)^2}\right] \qquad \sigma_t = 18.783\,ksi \qquad \text{(Eq. C3.1.2.1-9)}$$

Lateral-torsional buckling per C3.1.2.1(a)

$$F_e := \frac{C_b \cdot r_o \cdot A}{S_g} \cdot \sqrt{\sigma_{ey} \cdot \sigma_t} \qquad F_e = 26.064\,ksi \qquad \text{(Eq. C3.1.2.1-5)}$$

Note, for singly symmetric sections the x-axis is assumed to be the axis of symmetry.

$M_{cre} := S_g \cdot F_{cre}$ $\quad$ $M_{cre} = 57.291\,kip \cdot in$

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